Certifiable Pre-Play Communication: Full Disclosure ONLINE APPENDIX

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Weak Sequential Equilibria and Ex Post Masquerade

In this appendix, we develop a way to construct fully revealing equilibria by working directly with ex post masquerading payoffs, without having to aggregate them.By doing so, we can show existence of a fully revealing equilibrium when the ex post masquerading payoffs have increasing differences, regardless of the information structure. The idea is that to enforce full revelation, players can be skeptical by attributing messages that deviate from full revelation to a worst case type of the *ex post* masquerade relation. Indeed, if all players but i have revealed their type, the other players can condition their beliefs on t_{-i}. The existence of ex post worst case types is sufficient to get a fully revealing equilibrium independently of the specifics of the information structure. However, the caveat of this construction is that, in general, such beliefs violate one of the implications of strong sequential equilibria that we derived in Lemma 1. So the approach presented in this appendix sacrifices strong belief consistency and weakens the equilibrium concept used in the paper.

The Equilibrium Notion. We use the notion of weak sequential equilibria in the sense of Myerson (1991). They are defined as equilibria that satisfy sequential rationality and weak belief consistency. Weak consistency here means Bayesian consistency on the equilibrium path and off-path beliefs that are consistent with evidence.[1]

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[1] They correspond to what most papers call perfect Bayesian equilibria. Because this term is used in many different ways in the literature, we find it clearer to use the terminology of Myerson (1991). It is implied by strong consistency.

We first seek to provide sufficient conditions for the existence of fully revealing weak sequential equilibria. To do that, we construct equilibria with extremal beliefs such that off the equilibrium path beliefs following a unilateral deviation are homogeneous across non-deviators.[2] The equilibria we construct also satisfy Definition 4 of the main paper. To summarize, the equilibria that we construct are fully revealing weak sequential equilibria with homogeneous extremal beliefs that implement the selection $a^*(\cdot)$ on and off the equilibrium path.

Ex Post Masquerade Relation and Full Revelation. We start by adapting our definitions to the ex post treatment. For every t_{-i}, the ex post masquerade of player i given t_{-i} is the relation defined by $t_i \xrightarrow{\mathcal{M}(t_{-i})} s_i$ if and only if $v_i(s_i|t_i;t_{-i}) > v_i(t_i|t_i;t_{-i})$. The set of ex post worst case types of the set $S_i \subseteq T_i$ given t_{-i} is defined by $\text{wct}(S_i \mid t_{-i}) := \{s_i \in S_i \mid \nexists\, t_i \in S_i, t_i \xrightarrow{\mathcal{M}(t_{-i})} s_i\}$. We assume that, for every player i, the function $v_i(s_i|t_i;t_{-i})$ is lower semi-continuous in s_i.

Theorem 1 (Weak Sequential Equilibria)**.** *There exists a fully revealing weak sequential equilibrium with extremal and homogeneous beliefs that implements $a^*(\cdot)$ whenever the following conditions are satisfied for every i*

(i) For every t_{-i}, the set $M_i^{-1}(m_i)$ admits an ex post worst case type.

(ii) The correspondence $M_i(\cdot)$ admits an evidence base.

Proof. Pick an evidence base $\mathcal{E}_i$ for each player, and consider the strategy $e_i(\cdot)$ for each player in which i plays according to her evidence base mapping. By definition of an evidence base, this strategy profile is separating. Suppose that all players believe that the message $e_i(t_i)$ is sent by t_i only. Then the beliefs are consistent on the equilibrium path. Now consider a unilateral deviation $m_i \neq e_i(t_i)$ of player i of type t_i. If $m_i = e_i(s_i)$ for some $s_i \neq t_i$, this deviation cannot be beneficial, as other players will believe that m_i was sent by type s_i, which is a worst case type of $M_i^{-1}(m_i)$. Now suppose that $m_i \notin \mathcal{E}_i$, so m_i is an off-path message. Assume that the beliefs formed by other players after observing m_i puts probability 1 on a type $s_i^*(m_i, t_{-i}) \in \text{wct}(S_i \mid t_{-i})$. This is possible because all other players have sent an equilibrium message which is correctly interpreted as their true type, so all players know t_{-i}. This belief is an extremal belief that is consistent with the evidence contained in m_i. The interim payoff of player i if she sends m_i is therefore given by

$$E\left(v_i\big(s_i^*(m_i,t_{-i})|t_i;t_{-i}\big) \mid t_i\right) \leq E\left(v_i\big(t_i|t_i;t_{-i}\big) \mid t_i\right) = v_i(t_i|t_i),$$

where the inequality comes from the fact that $s_i^*(m_i, t_{-i})$ is an ex post worst case type. But this shows that m_i is not a profitable deviation and concludes the proof. □

[2] Note that, for weak sequential equilibria, Lemma 1 of the main paper no longer applies, so homogeneity is not imposed by the equilibrium concept coupled.

Ex Post Acyclic Masquerade Property. We say that a game satisfies the *ex post acyclic masquerade property* if, for every player i, and every t_{-i}, the ex post masquerade relation of i given t_{-i} is acyclic. The characterization of acyclic masquerade relations in Proposition 1 holds for the ex post masquerade if we condition each statement on t_{-i} and replace masquerade relation by ex post masquerade relation, and worst case type by ex post worst case type. It follows that, in the class of games with the ex post acyclic masquerade property, the existence of an evidence base for each player is a sufficient condition for the existence of a fully revealing weak sequential equilibrium. From Remark 1, we know that it is also necessary, so we have the following corollary:

Corollary 1. *Suppose that the ex post acyclic masquerade property is satisfied. Then there exists a fully revealing weak sequential equilibrium that implements $a^*(\cdot)$ if and only if there exists an evidence base for every player i.*

The sufficient conditions in Theorem 2 hold for the ex post acyclic masquerade property provided that the interim masquerading payoffs are replaced by the ex post masquerading payoffs. The following example uses the (ID) property of ex post masquerading payoffs. In this multiple senders example, we obtain existence of a fully revealing weak sequential equilibrium under mild assumptions on the preferences of the players, and no assumptions on the type distribution. To prove the existence of a fully revealing equilibrium that satisfies strong belief consistency by an aggregation result, we would have to either assume that types are independent and use Lemma 3 (iii) , or make some unnatural assumptions on the utilities and use a more sophisticated aggregation result.

Example 1 (Multiple Senders - Single Receiver Games)**.** One player with no private information, the receiver, wants to implement her ideal action $a^*(t) \in \mathbb{R}$. The partially and asymmetrically informed players, the senders, are indexed by i. T_i is a (possibly finite) compact subset of $\mathbb{R}$ endowed with its natural order. The assumption of lower semi-continuity of the ex post masquerading payoffs is ensured if, for every i, $u_i\big(a^*(s_i, t_{-i}), t_i, t_{-i}\big)$ is lower semi-continuous in s_i. Assume that:

(i) $a^*(\cdot)$ is non-decreasing.

(ii) For every sender i, the function $u_i(a, t_i, t_{-i})$ has increasing differences in (a, t_i).

Under these assumptions, $v_i(s_i|t_i; t_{-i}) = u_i\big(a^*(s_i, t_{-i}), t_i, t_{-i}\big)$ has increasing differences in (s_i, t_i), and therefore the ex post acyclic masquerade property is satisfied. To see that, take $s_i' \succ s_i$ and $t_i' \succ t_i$ and note that

$$v_i(s_i'|t_i'; t_{-i}) - v_i(s_i|t_i'; t_{-i}) = u_i\big(a^*(s_i', t_{-i}), t_i', t_{-i}\big) - u_i\big(a^*(s_i, t_{-i}), t_i', t_{-i}\big)$$
$$\geq u_i\big(a^*(s_i', t_{-i}), t_i, t_{-i}\big) - u_i\big(a^*(s_i, t_{-i}), t_i, t_{-i}\big) = v_i(s_i'|t_i; t_{-i}) - v_i(s_i|t_i; t_{-i}),$$

where the inequality comes from the fact that $a^*(s_i', t_{-i}) \geq a^*(s_i, t_{-i})$ by (i), and from (ii). Therefore, there exists a fully revealing weak sequential equilibrium as long as we have an evidence base for every player. $\diamond$

Applications. This approach allows us to extend all the results that rely on type independence in the applications section of the main paper to any (full support) type distribution, provided that we consider weak sequential equilibria instead of strong sequential equilibria.

References

Myerson, R. B. (1991): *Game Theory, Analysis of Conflict*, Harvard University Press.