Information and Incentives in A Model of Contest Between Large Groups *

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Abstract

This paper studies a model of social contest between two large groups, in which individual agents can choose whether or not to actively support their group. They become active supporters if they expect their group to win with a sufficiently high probability. The identity of the winner is decided according to a deterministic social rule that is a function of the groups' strengths and activity rates. Agents have imperfect and heterogeneous information, from both public and private sources with known precisions, about the strength of the other group. No modification of the information structure gives an unambiguous advantage to one particular team. The effects of private and public precisions on equilibrium outcomes are always opposed. Increasing the precision of the public information of a group has the same effect as increasing the sensitivity of the social rule to its activity rate, illustrating the idea that public information favors collective action. For a particular example of the social rule, the paper characterizes the information structure that would arise endogenously in two different settings: a disclosure game between group leaders and a contest design.

1 Introduction

Many economic environments have been described as contests or all-pay auctions, including R&D races, lobbying, elections, rent seeking. Contests have also been proposed as a way to provide incentives in the workplace. The common feature of these situations is that they involve agents incurring a cost in order to compete for a prize. The contestants are generally modeled as individual decision makers. However, in many real situations they are groups (firm, political party, special interest group, R&D teams) of individuals who each have some input in the action

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of the group. These groups may be organized in different ways, and decision-making within these groups can be more or less centralized and more or less cooperative, with an impact on the way the group plays. This paper proposes a model of contest between two teams, in the case of incomplete information environments with groups where decision-making is completely decentralized and non-cooperative.

The paper focuses on the coordination problem between agents within each team and on the impact of the information structure. If the probability of winning the contest for a team increases with the aggregate effort of its members, there are strategic complementarities between agents. If, for instance, individuals have to choose between a low and a high level of effort, they will be more inclined to choose the high level if their teammates choose the high level as well. It is well known that in these conditions, the aggregate effort of a team is sensitive to the fine structure of its members' beliefs prevailing because an agent has more incentives to contribute to the collective effort if she is confident that her team has good chances to get the prize, and that other agents in her team are also confident about it, and are confident about their teammates being confident, and so on. With incomplete information, and a rich information structure, the modeler, or prominent players in the model, have several levers to influence the beliefs of the team members. For example, if each team has imperfect information about the strength of the other teams. Is a team more likely to win the contest when it has very precise information about its opponents? Or is ignorance a more effective way to ensure high levels of contribution? Does a more homogeneously informed group stand more chances to win? What is the best information structure from the point of view of a contest designer who wants to maximize the total contribution? What information structures emerge if they are determined by the strategic choices of some individual team leaders?

This paper provides a framework in which these questions can be analyzed and at least partially answered. It is strongly related to a recent theoretical literature[1] that has focused on the problem of coordinating expectations of populations of agents, and on the effects of

[1] In particular Morris and Shin (2002b), Hellwig (2005), Angeletos and Pavan (2007).

public and private information. For example, Morris and Shin (2007) gives an account of how central bankers have come to understand their role as one of coordinating the expectations of economic agents through the communication of public information. This paper takes the view that other prominent institutions play such a coordinating role. Political parties, for example seek to coordinate the beliefs of their supporters in order to raise campaign contributions. In this case the problem is not merely one of coordinating agents, but of coordinating them on the right action from the point of view of the party (contributing). In an electoral contest, a party should also try to coordinate the supporters of the other party on a more passive action (no contribution). Lobbying by special-interest groups and advocacy wars can be analyzed very similarly. Another example is the design of R&D policies or workplace incentives. It has been argued that contests give incentives to expend effort. However, it is important, in order to obtain this effect, to coordinate the expectations of the members in each team so that they believe they can win with a sufficiently high probability or they may consider that it is not worth trying.

The model is highly stylized and is meant to illustrate possibly interesting trade-offs in the provision of information. No exogenous cost of communication or information is assumed and all trade-offs arise endogenously. It is a contest game between two teams consisting each in a continuum of agents. A team is characterized by its strength. Individuals face the binary choice of whether to be active in their group, where being active can imply things such as exerting effort, contributing financially, and simply acknowledging support for one's group (this may entail some reputation or opportunity costs for example). The aggregate action of a team is identified with the proportion of its members who are active. The outcome of the contest (*i.e.* the identity of the winner) is decided according to a deterministic social rule[2] that favors higher "strength"[3] and activity rates. Payoffs are such that agents are willing to be active if their estimated probability of winning the contest is sufficiently high, which is obtained by

[2]That can be understood as a reduced form for a more complex process such as an election.

[3]Strength is in fact defined by the social rule itself and does not have any other intrinsic meaning, it is assumed to be payoff irrelevant *ex post*.

assuming that being active is rewarding in case of victory and costly otherwise. Agents know the strength of their team but have imperfect information about the other team, both from private and public sources with known precisions, so that the *ex interim* beliefs are heterogeneous within groups. This framework makes it possible to study the effects of information structures (*i.e.* the precisions) on equilibrium outcomes.

The first part of the paper is concerned with the existence and uniqueness of equilibria. I start with equilibria that satisfy a certain monotonicity property (that if a group wins with a certain strength, it also wins when its strength is increased), and Proposition 1 characterize the strategies used in these equilibria in closed form, up to one function (the *frontier function*) that determines the pivotal pairs of strength parameters (it gives the level of strength needed by one group to overcome a given level of strength of the other group in equilibrium). Theorem 1 provides a functional equation as well as a necessary condition to be satisfied by the equilibrium *frontier function*, andTheorem 2 gives a sufficient (and almost necessary) condition on the primitives of the model for the existence of such an equilibrium. The latter also shows that this equilibrium is unique in its class whenever the sufficient condition for existence is satisfied. Finally Theorem 3 shows that the equilibrium is unique among all equilibria under an additional assumption on the primitives. The assumption is an analog for the present framework of the dominance region condition in the global games literature. It requires the existence of regions in the space of strength parameters such that one of the groups wins regardless of the actions played.

Some comparative statics results are summarized in Proposition 3. Interpretations for these results are provided through a series of remarks. In particular, no modification of the information structure can strictly expand the winning set of a group, that is the set of strength parameters such that this group is the winner. On the other hand, a change in the payoff structure, or in the realization of the public signals, always strictly expands the winning set of one group, and shrinks that of the other one. The direction of all these effects changes depending on whether the private precisions are high or low relative to the public precisions. This is most striking for the public signals and the payoff parameters. With relatively precise private information, higher public signals about the strength of the opponent induce smaller winning sets, but the opposite is true when public information is very precise. An intuition is that when public information is precise, players become more aggressive as they learn that their opponent is strong because they believe that their teammates will do the same. Conversely, when public information is not reliable, players do not trust their teammates and shy away. The same is true with incentives. Increasing the difference between the payoffs of active and passive agents in case of victory and decreasing it in case of defeat naturally expands the winning set of a group if the private precision is relatively high. However, if public information is sufficiently precise relative to private information, this natural intuition no longer holds, and the result of that change in payoffs is actually to shrink the winning set of the group. This offers an explanation of the casual observation that some groups with seemingly poor incentives or very bad news about their chances can actually do better. The intuition is again that bad public news or bad incentives make individuals more aggressive with respect to their private signals when these are relatively unprecise with respect to the public ones. Another interesting result (Proposition 4) is that increasing the precision of the public information of one group has the same effect as increasing the sensitivity of the social rule to the collective action of that group. This result formalizes neatly the idea that public information favors coordination.

Finally, the last part of the paper considers a particular case that makes closed form calculations particularly simple, yielding an expression for the *ex ante* probability to win implied by the equilibrium. These formulas are used to provide *ex ante* comparative statics in Proposition 5 showing the effects of public precisions on the probabilities of winning. I proceed to analyze two decision problems played at the *ex ante* stage that endogenize the information stucture. In the first setting, two team leaders commit *ex ante* to the precision of the information they will disclose to the other team upon learning their own strength. Their objective is to maximize the probability that their group wins the contest, and their decision gives rise to a game which is analyzed in Proposition 6. The second setting considers the problem of a contest designer who designs rules of public disclosure of information across groups in order to maximize the total activity rate of the agents (Proposition 7).

2 Related Literature

From a methodological perspective, this paper is related to the literature on games of regime change and provides a natural extension of this literature. In these models, a continuum of agents decide whether to attack an institution, whose ability to defend itself depends on the size of the attack and on the fundamentals of the economy (that play a role that is analogous to that of the strength parameter of the other team in our model), which is imperfectly observed by the agents. The game is sequential in that the institution plays after having observed the size of the attack. In our model, each team is similar to the continuum of agents in the currency attack model. However all agents play simultaneously. But when the strategy profile of one of the teams is fixed, and assumed to depend on the parameters with appropriate monotonicities, the best response problem of the other team looks exactly like the coordination game analyzed by Morris and Shin. Therefore the global game information structure can be used to shrink the best response range of each team and give it some structure much in the same way as it is used for equilibrium selection in the coordination games. This also suggests that the ideas underlying this paper can in principle be extended to more general models with more than two teams, and with many other generalizations (finite number of players in each team, larger action sets, more general information structures) that have proved tractable for global coordination games as in Morris and Shin (2002a), and Frankel, Morris and Pauzner (2003).

The paper is also related to the literature on group play, *i.e.* games played between and by groups. Duggan (2001) defines "group Nash" equilibrium as a solution concept that is non-cooperative across groups and cooperative within groups. Charness and Jackson (2007) take their observations of an experiment on a Stag Hunt game played by groups of two as a starting point to build a new solution concept called *robust-belief equilibrium*. However, most of the literature on group play is experimental and Bornstein (2008) provides an interesting survey. In particular, Bornstein (2008) introduces a classification of the games literature according to the nature of the players (Nature, Individual, Unitary groups, and non-cooperative Groups) and argues that some cells have been little explored such as the G-G cell (non-cooperative groups versus non-cooperative groups) into which the model of this paper falls.

3 The model

3.1 Setup

Players, Actions, and Outcomes. There are two teams $g \in \{-1, 1\}$ of players consisting each of a continuum (a non-atomic space $(I, \mathcal{I}, \lambda)_g$ with finite measure normalized to 1) of agents. Agents are indexed by $i \in I_g$. In each group, agents decide whether to be active $a_i \in A = \{0, 1\}$. The *activity rate* of group g is defined as the measure of the set of active agents $l_g = \lambda_g\big(\{i \in I_g | a_i = 1\}\big)$. The outcome of the contest is the identity of the winning group, and I also allow for a neutral outcome 0 in which the teams are tied. The outcome space is denoted by $\Omega = \{-1, 0, 1\}$.

Social Rule. The outcome of the contest is decided according to a social rule contingent on the *activity rates* of the groups, and a state of the world $\theta = (\theta_{-1}, \theta_1) \in \mathbb{R}^2$ that captures the uncertainty of the agents about the bias of the social rule. The bias of the rule towards group g is assumed to be increasing in θ_g and decreasing in θ_{-g}. Hence, θ_g is to be interpreted as a *strength* parameter for group g. The social rule is also assumed to favor higher activity rates.

Assumption 1 (Smooth Social Rule). *The social rule is fully described by a continuously differentiable function $R : \mathbb{R}^2 \times [0, 1]^2 \to \mathbb{R}$ such that:*

(i) The outcome of the contest is given by the sign of $R(l_1, l_{-1}, \theta_1, \theta_{-1})$: team 1 wins if $R(l_1, l_{-1}, \theta_1, \theta_{-1}) > 0$, and team -1 wins if $R(l_1, l_{-1}, \theta_1, \theta_{-1}) < 0$. Otherwise, the teams are tied.

(ii) R is strictly increasing in l_1 and θ_1 and strictly decreasing in l_{-1} and θ_{-1}.

R can be interpreted as the *ex post* bias of the social rule towards group 1. To make notations symmetric, I define the functions $R_1 = R$ and $R_{-1} = -R$, so that R_g denotes the *ex post* bias towards g. The sign function is a mapping from the real line to the set $\{-1, 0, 1\}$ that maps any negative number to -1 and any positive number to 1. For some results, it will be useful to consider that a social rule satisfies the following property.

Definition 1 (Dominance Regions **(DR)**). *The social rule has dominance regions if for every $\theta_1 \in \mathbb{R}$ there exists $\overline{\psi}(\theta_1)$ and $\underline{\psi}(\theta_1)$ in $\mathbb{R}$ such that $R\big(0, 1, \theta_1, \underline{\psi}(\theta_1)\big) = R\big(1, 0, \theta_1, \overline{\psi}(\theta_1)\big) = 0$.*

This assumption means that, conditional on any realization of g's strength parameter θ_g, there exists a state of the world where g wins regardless of the *activity rates*, and a state of the world where g loses regardless of the *activity rates*. It is a dominance region assumption because if the players of a group are sure to be in a region of the strength space in which they can win the contest regardless of activity rates, it is a dominant strategy for them to be active and reap the benefits of a certain victory.

Payoffs. The *ex post* payoff of a player depends only on her own action and the outcome of the game. Hence the payoff function of a player $i \in I_g$ is a function $v_{i,g} : A \times \Omega \to \mathbb{R}$. The benefit of being active for player $i \in I_g$ given an outcome ω is $\Delta_{i,g}(\omega) = v_{i,g}(1, \omega) - v_{i,g}(0, \omega)$. It is assumed to be strictly positive when $\omega = g$ and strictly negative otherwise, so that being active is strictly profitable in case of victory, and strictly detrimental in any other case. This has the effect of assuming free-riding away, and it supposes that the group is able to detect free-riders with a strictly positive probability. For notational simplicity, I assume that the payoff in case of neutral outcome is the same as the payoff in case of defeat, and let $\neg g$ denote the event $\{0, -g\}$. The *cost of activity* is

$$\gamma_g = \frac{-\Delta_{i,g}(\neg g)}{\Delta_{i,g}(g) - \Delta_{i,g}(\neg g)} \in (0, 1),$$

where the index i has been omitted because I make the assumption that this cost parameter is homogeneous within groups. It captures the (dis)incentive of the player to become active. Intuitively, the higher γ_g, the less players in group g want to become active. This parameter can be used as a policy variable by a group leader. For example, for given payoffs to active and inactive members, a higher ability of the group to recognize the ones from the others in case of victory would lower γ and increase the incentive to become active.

Information Structure. The strength parameters are initially drawn independently from a normal distribution with mean 0 and precision $\bar{P}$. This state of information is referred to as the *ex ante stage* in the rest of the paper.

An agent i in team g is perfectly informed about the strength of its own team θ_g, and receives a noisy private signal $x_{i,g}$ and a noisy public signal y_g about the strength of the other team θ_{-g}. The public signals are fully public, meaning that each of them is observed by the agents of both teams. The signalling technology is defined as follows

$$\text{Private Signals:} \qquad x_{ig} = \theta_{-g} + p_g^{-1/2}\varepsilon_i \ , \quad \varepsilon_i \sim \mathcal{N}(0,1), \tag{1}$$

$$\text{Public Signals:} \qquad y_g = \theta_{-g} + P_g^{-1/2}\eta_g \ , \quad \eta_g \sim \mathcal{N}(0,1), \tag{2}$$

where all the noise terms are independent from one another. p_g is the *precision of private information* of team g and P_g is the *precision of public information* of team g. This state of information is the *ex interim stage*. Bayesian rationality implies that the interim beliefs of an agent $i \in g$ about θ_{-g} upon receiving signals x and y are given by a normal distribution with mean

$$m_g(x,y) = \frac{p_g}{\Pi_g}x + \frac{P_g}{\Pi_g}y, \tag{3}$$

and precision

$$\Pi_g = \bar{P} + P_g + p_g.$$

Π_g is called the *total accuracy of information.* $m_g(x,y)$ is the *best forecast*[4] of an agent in team

[4]The one that minimizes expected errors.

g with signal (x, y).

The expected payoff of an agent $i \in g$ with private signal x and public signal y according to her *ex interim* beliefs is

$$\mathcal{P}^e_{i,g} = a_i\Big(\big(v_{i,g}(1,g) - v_{i,g}(1,\neg g)\big)\Pr\big(\omega = g|x,y\big) + v_{i,g}(1,\neg g)\Big) \\ + (1-a_i)\Big(\big(v_{i,g}(0,g) - v_{i,g}(0,\neg g)\big)\Pr\big(\omega = g|x,y\big) + v_{i,g}(0,\neg g)\Big), \tag{4}$$

where the event $\neg g$ means that either $-g$ wins or the neutral outcome is reached.

Note that the distinction between public-signals-updated priors and priors plays no role in the equilibrium analysis to follow. The distinction is introduced at this stage in order to study the specific effects of public information once equilibrium behavior is pinned down.

3.2 Examples

Example 1 (A model of Elections). Suppose that the population is divided into three groups: a large group of non partisans of size N, and two groups of partisans of either 1 or -1, with size $N_g < N$. The quality of group g's candidate is θ_g, and I assume that the probability that a non-partisan votes for g is equal to $\exp(\theta_g)/(\exp(\theta_g) + \exp(\theta_{-g}))$. The fraction of partisans of group g that decide to support their group is given by l_g. Partisans may decide not to support their group, because *ex post* it is harmful for them to do so if their party is not elected. The score of party g in the election is then given by

$$N\frac{\exp(\theta_g)}{\exp(\theta_g) + \exp(\theta_{-g})} + N_g l_g,$$

and the contest gives rise to the social rule function

$$R_g = \big(N + N_g l_g - N_{-g} l_{-g}\big)\exp(\theta_g) - \big(N + N_{-g} l_{-g} - N_g l_g\big)\exp(\theta_{-g}).$$

It is easy to see that this social rule satisfies the smoothness assumption of Assumption 1 and (DR).

Example 2 (Output Contest). This can be a setup for the R&D contest model. Two competing technologies or theories are described by a technological parameter $A_g = \exp(\theta_g)$. The number of contributions by an R&D team is l_g. The output or total quality of contribution of group g is given by $A_g \exp\left(\Phi^{-1}(l_g)\right)$. The social rule just compares the outputs of the two teams. This gives rise to a smooth social rule that does not satisfy (DR) $R_g = \theta_g + \Phi^{-1}(l_g) - \theta_{-g} - \Phi^{-1}(l_{-g})$. This example is particularly useful because it yields very simple close- form calculations.

Example 3 (Raising Contributions). The population is composed of a group of non-partisans of size N and two groups of partisans of size $\exp(\theta_g)$. Partisans decide whether to finance their group. Contributions are fixed in size, so the total contribution for g is proportional to l_g. A non-partisan has a probability $l_g/(l_g + l_{-g})$ to vote for g. Hence the score of group g can be defined as $\exp(\theta_g) + N\frac{l_g}{l_g+l_{-g}}$, the total number of votes in favor of g assuming that partisans always vote for their party, even if they did not contribute. Then the social rule is smooth and satisfies ((DR)).

4 Equilibrium Analysis

4.1 Definitions

The full information type of an agent $i \in I_g$ is a vector $(\theta_g, y_g, y_{-g}, x_i)$. Since the public signals are both observed by all the agents of both groups, I can omit them in the description of the informational type and let the type of agent i be (θ_g, x_i). A pure strategy for agent i is a mapping $a_i : \mathbb{R}^2 \to A$ from her type space to the action space. A strategy profile for group g is completely described by a function $a_g : \mathbb{R}^2 \times I_g \to A$ such that $a_g(\theta_g, x_i, i)$ is the action taken by agent i upon receiving a private signal x_i. In the following, it will be useful to consider the function $a_g(\theta_g) = a_g(\theta_g, ., .)$. A full strategy profile is a pair of functions $\pi = (a_g, a_{-g})$.

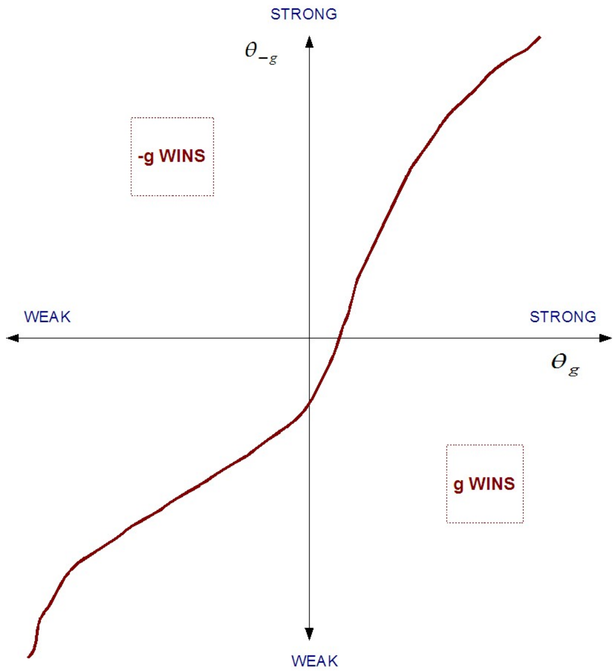

Figure 1: **Outcomes on the Type Space**

Given a strategy profile π, an informal law of large numbers[5] over the set of agents I_g implies that the probability that an agent with signal x participates is given by

$$\rho_g^\pi(x; \theta_g, y_g, \gamma_g) = \int_{I_g} a_i(x) d\lambda_g(i).$$

And I can define the *team strategy* of team g as

$$l_g^\pi(\theta_g, \theta_{-g}) = \int_{-\infty}^{\infty} \rho_g(\theta_{-g} + \varepsilon; \theta_g, y_g, \gamma_g) d\Phi(\varepsilon), \tag{5}$$

where $\Phi(.)$ is the cdf of the standard normal distribution.

Therefore, a strategy profile π induces a new bias function R^π on the space of strength

[5]See Judd (1985) for a formulation of the issue of casually applying a law of large number to a continuum of random variables, and Hammond and Sun (2006) and Sun (2006) for a proper way of doing it, and a justification of its use in this paper. Note that the issue is generally ignored in the literature. Duffie and Sun (2004) and **?** are useful additional readings.

parameters defined by

$$R^{\pi}(\theta_1, \theta_{-1}) = R\Big(l_1^{\pi}(\theta_1, \theta_{-1}), l_{-1}^{\pi}(\theta_{-1}, \theta_1), \theta_1, \theta_{-1}\Big).$$

The induced bias function R^{π} is (strictly) *monotonic* if it is (strictly) increasing in θ_1 and (strictly) decreasing in θ_{-1}. It seems natural to expect equilibrium strategy profiles that partition the space of strength parameters in two regions such that each group wins when it is rather strong and the other group rather weak (Figure 1). A way to formalize this idea is through the following definition.

Definition 2 (Strictly Monotonic Single Crossing Property)**.** *R^{π} satisfies **(SMSCP)** if there exists a strictly increasing continuous function $\psi_1 : \overline{\mathbb{R}} \to \overline{\mathbb{R}}$ such that*

$$sign\Big(R^{\pi}(\theta_1, \theta_{-1})\Big) = sign\Big(\psi_1(\theta_1) - \theta_{-1}\Big).$$

The definition further assumes that the two regions can be separated by a strictly increasing continuous function in the space (θ_1, θ_{-1}) such that group -1 wins in the region above the graph of that function and group 1 wins in the region below that graph. Note that when (SMSCP) is satisfied, I can define the function $\psi_{-1} = \big(\psi_1\big)^{-1}$ that plays a symmetric role in the space (θ_{-1}, θ_1). In what follows, any of the ψ functions are referred to as a *frontier function* since the graph of either of them defines the frontier between the winning regions of the two groups. In looking for equilibria, I start by focusing on equilibria that satisfy (SMSCP) before considering other equilibria. Note that these equilibria form a superset of the class of monotonic equilibria.

Definition 3 (Monotonic Equilibrium)**.** *The players are said to use monotonic strategies if they use strategies that are increasing in their group type θ_g and decreasing in their individual type x_i. A monotonic equilibrium is a Bayesian Nash equilibrium in which agents use monotonic strategies.*

Lemma 1. *A monotonic equilibrium of the group contest game satisfies (SMSCP).*

Proof. See Appendix A. □

4.2 Characterization

A simple manipulation of the expected payoff equation (4) shows that it is optimal for a player i in group g to be active if and only if her estimated probability of winning (given her informational type) is strictly higher than her payoff parameter γ_g. Assuming that the other players are playing according to a strategy profile that satisfies (SMSCP)[6] the probability that g wins according to i is given by

$$\Pr\big(\theta_{-g} < \psi_g(\theta_g) | x, \theta_g\big) = \Phi\Bigg(\Pi_g^{1/2}\big(\psi_g(\theta_g) - m_g(x,y)\big)\Bigg). \tag{6}$$

Therefore a few calculations yield the following proposition that gives a formula for the strategies used by the players in an equilibrium in strategy profiles that satisfy (SMSCP).

Proposition 1. *In an equilibrium that satisfies (SMSCP), the players become active if their best forecast $m_g(x,y)$ of the strength parameter of the other group is strictly below a threshold*

$$\hat{m}_g = \underbrace{\psi_g(\theta_g)}_{\textit{Pivotal Strength}} \underbrace{-\Pi_g^{-1/2}\Phi^{-1}(\gamma_g)}_{\textit{Cost Recovery Term}}. \tag{7}$$

The activity rates are then given by

$$l_g(\theta_g, \theta_{-g}) = \Phi\big(\hat{\varepsilon}_g(\theta_g, \theta_{-g})\big), \tag{8}$$

where

$$\hat{\varepsilon}_g(\theta_g, \theta_{-g}) = p_g^{-1/2}\big(\Pi_g\psi_g(\theta_g) - p_g\theta_{-g} - \Pi_g^{1/2}\Phi^{-1}(\gamma_g) - P_g y_g\big). \tag{9}$$

Proof. See Appendix A. □

[6] Because of the continuum assumption, no deviation of player i can cause the full strategy profile not to satisfy (SMSCP).

The proposition shows in particular that in these equilibria, players use monotonic strategies. The threshold best forecast $\hat{m}_g$ that prevails in group g is equal to the pivotal strength of the opponent $\psi_g(\theta_g)$ (*i.e.* the value of θ_{-g} at which the outcome changes, given θ_g) plus a term arising from the need to recover the *cost of activity* γ_g. Note that the information structure affects both the threshold best forecast of the group, and the way agents form their best forecast as apparent in equation (3). Finally, the term $\hat{\varepsilon}_g$ is the threshold noise term corresponding to the threshold best forecast $\hat{m}_g$. For given strength types, the agents of group g that become active are those who receive a signal with a realized noise term below $\hat{\varepsilon}_g$.

To obtain a full characterization of the equilibria that satisfy (SMSCP), I need to characterize the equilibrium *frontier functions* $\left(\psi_g\right)_{g=-1,1}$. This is done in the following theorem, through an equation that defines the frontier function implicitly. The equation is a consequence of the fact that a pair of strength parameters (θ_g, θ_{-g}) that lie on the frontier must at the same time satisfy $\theta_{-g} = \psi_g(\theta_g) = \left[\psi_{-g}\right]^{-1}(\theta_g)$, and make the bias in the social rule equal to 0.

In order to state the theorem, I introduce some preliminary notations. Let

$$\tilde{R}_g\big(.,.,.,.\big) = R_g\big(\Phi(.), \Phi(.), ., .\big), \tag{10}$$

and

$$\alpha_g = p_g^{-1/2}\left(\bar{P} + P_g\right), \tag{11}$$

$$\beta_g = p_g^{-1/2}\big(P_g y_g + \Pi_g^{1/2}\Phi^{-1}(\gamma_g)\big). \tag{12}$$

Note that $\tilde{R} : \mathbb{R}^4 \to \mathbb{R}$ is just a rescaling of the social rule function $R : [0,1]^2 \times \mathbb{R}^2 \to \mathbb{R}$ and that it is still continuously differentiable.

The theorem also gives a (tautological) condition for the existence of an equilibrium that satisfies (SMSCP) which is simply that the equation that implicitly defines the frontier can be solved and that the solution is as required by (SMSCP).

Theorem 1. *The group-contest game admits an equilibrium that satisfies (SMSCP) if and only* *if*

(i) For every $\theta \in \mathbb{R}$, there exists a solution $\psi(\theta) \in \bar{\mathbb{R}}$ to the equation in ψ

$$\tilde{R}_g\Big(\alpha_g\psi - \beta_g \ , \ \alpha_{-g}\theta - \beta_{-g} \ , \ \theta \ , \ \psi\Big) = 0. \tag{13}$$

(ii) The function ψ thus defined is continuous and strictly increasing.

And in this case, the equilibrium frontier is defined by $\psi_g(\theta_g) = \psi(\theta_g)$.

Proof. See Appendix A. □

Note that in condition (i), the equation is stated for either of the two groups, but this does not matter since when the equation admits a solution for one group and (ii) is satisfied, the equation stated for the other group must admit a solution. This would not be true if the solution was not required to be strictly increasing, and hence invertible.

4.3 Existence and Uniqueness

As formerly noticed, Theorem 1 gives a tautological condition for existence, and a sufficient condition for existence that can be checked easily on the primitives of the model would be more interesting. Equation (13) suggests the use of an implicit function theorem. However, while the usual implicit function theorems are local, I need a solution $\psi(.)$ to equation (13) that is defined on the whole real line. The following global implicit function theorem from Zhang and Ge (2006) provides us with the appropriate mathematical tool

Lemma 2. *Assume that $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ is a continuous mapping and that it is continuously differentiable in the second variable $u \in \mathbb{R}^m$. If*

$$\left|\left[\frac{\partial}{\partial u} f(x,u)\right]_{ii}\right| - \sum_{j\neq i}\left|\left[\frac{\partial}{\partial u} f(x,u)\right]_{ij}\right| \geq d, \qquad \forall (x,u) \in \mathbb{R}^n \times \mathbb{R}^m, \quad i = 1, \cdots, m, \tag{14}$$

for a fixed constant $d > 0$, then there exists a unique mapping $g : \mathbb{R}^n \rightarrow \mathbb{R}^m$ such that $f(x, g(x)) = 0$. Moreover, this mapping g is continuous. Additionally, if f is continuously differentiable, then the obtained g is also continuously differentiable.

Proof. The Lemma is stated in this form and proved in Zhang and Ge (2006)[Theorem 1]. A version with $m = 1$ and without the last result is proved in Ge and Wang (2002) [Lemma 1]. $\square$

The condition in equation (14) is a dominant diagonal condition on the matrix of derivatives of f with respect to u. For our problem, I only need the case $m = 1$. The application of this result provides us with a sufficient condition on the partial derivatives of $\tilde{R}_g$ for the existence of an equilibrium. The sufficient condition has two interpretations. It can be understood as a requirement that the frontier function $\psi(.)$ solves equation (13) and is strictly increasing, or as a requirement that the frontier function solves equation (13) for a certain g and that its inverse solves the same equation stated for the other group $-g$. The sufficient condition as stated also implies uniqueness. Each function $\tilde{R}_g$ is a continuously differentiable function (by assumption) of four real variables and I denote its k-th partial derivative by $\partial_k \tilde{R}_g$.

Theorem 2. *Suppose that there exists a constant $d > 0$ such that for every $(\theta, \psi) \in \mathbb{R}^2$, it holds that*

$$\Big[-\big(\alpha_g \partial_1 + \partial_4\big) \cdot \big(\alpha_{-g} \partial_2 + \partial_3\big)\Big] \tilde{R}_g \Big(\alpha_g \psi - \beta_g \ , \ \alpha_{-g}\theta - \beta_{-g} \ , \ \theta \ , \ \psi\Big) \geq d. \tag{15}$$

Then there exists a unique equilibrium in strategy profiles that satisfy (SMSCP). Furthermore, the frontier function ψ_g is then continuously differentiable in θ_g, and in the parameters on any open set of the parameters where the condition of equation (15) is satisfied.

Proof. See Appendix A for a detailed proof. The argument is outlined below. $\square$

In the notations of the theorem, ∂_k is an operator and I use natural operations on operators to form other operators. Hence $\Big[-\big(\alpha_g \partial_1 + \partial_4\big) \cdot \big(\alpha_{-g} \partial_2 + \partial_3\big)\Big]$ must be read as an operator applied to the function $\tilde{R}_g$ to obtain a new function evaluated at the point $\Big(\alpha_g\psi - \beta_g \ , \ \alpha_{-g}\theta - \beta_{-g} \ , \ \theta \ , \ \psi\Big)$. Note that when the condition of equation (15) is satisfied, the frontier function ψ is continuously differentiable and, by implicit differentiation, its derivative with respect to θ is given by

$$\psi'(\theta) = \left[-\frac{\alpha_{-g}\partial_2 + \partial_3}{\alpha_g\partial_1 + \partial_4}\right]\tilde{R}_g\Big(\alpha_g\psi(\theta) - \beta_g \ , \ \alpha_{-g}\theta - \beta_{-g} \ , \ \theta \ , \ \psi(\theta)\Big), \tag{16}$$

and must be strictly positive. Because the condition also implies, by Lemma 2, that there exists a solution ψ to equation (13) I obtain the desired conclusion that the solution exists and is strictly increasing.

It is important to notice that the condition in Theorem 2 bears on all the primitives of the game *i.e.* both the social rule function and the information structure. It is possible for example that with a given social rule, an equilibrium exists (and is unique) for a certain specification of the information structure but not for another one (see Section 5.2 where the particular case of Example 2 is treated). This is a significant restriction when doing comparative statics with respect to the information structures. The condition is also quite restrictive since it has to hold uniformly.

Holding the information structure fixed, the condition on the partial derivatives says that the ratio of the marginal effect of the *activity rate* of group g on the social rule function over the effect of the strength of group $-g$ must be either high for both $g = -1, 1$ or low for both. Holding these marginal effects fixed, the condition says that the relative precision of the private signal (that is the ratio between private and public precision) must be either high for both groups or low for both groups. The condition also suggests that increasing the sensitivity of the social rule to the collective action plays the same role as increasing the relative precision of the public signals, which is coherent with the natural intuition that better public information helps coordination. The intuitions mentioned in this paragraph find a formal expression in two results in the remainder of the paper (Proposition 2 and comments, and Proposition 4). The condition can also be related to symmetry. When both the information structures and the social rule are symmetric, an equilibrium always exists. When either the information structures or the social rule becomes asymmetric for the two groups, equation (15) cannot be satisfied.

Finally, the condition is almost necessary since in order to satisfy the strictly increasing frontier condition, a continuously differentiable solution to equation (13) would have to satisfy that, for every $(\theta, \psi) \in \mathbb{R}^2$,

$$\Big[-\big(\alpha_g \partial_1 + \partial_4\big) \cdot \big(\alpha_{-g} \partial_2 + \partial_3\big)\Big] \tilde{R}_g \Big(\alpha_g \psi - \beta_g \ , \ \alpha_{-g} \theta - \beta_{-g} \ , \ \theta \ , \ \psi\Big) > 0. \tag{17}$$

Theorem 2 also proves uniqueness in the class of equilibria that satisfy (SMSCP). The next theorem proves that this equilibrium is the unique equilibrium of the group contest game when the condition of Theorem 2 is satisfied and the social rule satisfies the dominance region condition (Definition 1). The proof relies on iterated deletion of strictly dominated strategies as in Milgrom and Roberts (1990), even though the payoffs of the game are not generally supermodular.

Theorem 3. *If the social rule function satisfies (DR) and the condition in Theorem 2, the group contest game admits a unique equilibrium. In this case, the equilibrium strategy profile is the unique profile that survives iterated deletion of strictly dominated strategies.*

Proof. See Appendix A. □

Note that the result is not true in general for social rules that do not satisfy (DR) as illustrated by Example 2. For this example, it is always an equilibrium of the game that all the agents in one group decide to be active while all the agents in the other group decide to be passive. In particular, it is also the case when there exists an equilibrium in strategy profiles that satisfy (SMSCP).

The last result of this section gives a simple sufficient condition on the partial derivatives of $\tilde{R}_g$ for an equilibrium to exist when private precisions in both groups are sufficiently high relative to public precisions, and another one such that an equilibrium exists when public precisions in both groups are sufficiently high relative to private precisions. If the first result is clearly reminiscent of the literature on global games, the second one is more surprising since precise public information and imprecise private information tend to generate multiple equilibria in this literature (see Hellwig (2002)).

Proposition 2. *An equilibrium of the group contest game that satisfies (SMSCP) exists whenever either of the following holds:*

(i) There exists $m > 0$ and $\eta > 0$ such that $\frac{\partial_1 \tilde{R}_g(z)}{\eta - \partial_4 \tilde{R}_g(z)}$ and $\frac{-\partial_2 \tilde{R}_g(z)}{\eta + \partial_3 \tilde{R}_g(z)}$ are both bounded below by m for every $z \in \mathbb{R}^4$, and the information structure satisfies $p_g^{1/2} \leq m\left(\bar{P} + P_g\right)$ for every g.

(ii) There exists $M > 0$ and $\eta > 0$ such that $\frac{\partial_1 \tilde{R}_g(z)}{-\eta - \partial_4 \tilde{R}_g(z)}$ and $\frac{-\partial_2 \tilde{R}_g(z)}{-\eta + \partial_3 \tilde{R}_g(z)}$ are both bounded above by M for every $z \in \mathbb{R}^4$, and the information, and the information structure satisfies $p_g^{1/2} \geq M(\bar{P} + P_g)$ for every g.

Proof. See Appendix A. □

In particular, when both boundedness assumptions are satisfied, equilibria exist both in high and low relative private precision regions.

5 Comparative Statics

5.1 General Results

The fact that the frontier functions are continuously differentiable in the parameters when they exist makes the comparative statics exercise relatively easy by implicit differentiation. On any connected open set of the parameters such that the condition of Theorem 2 is satisfied, there are two possibilities:

(i) $\left(\alpha_g\partial_1 + \partial_4\right)\tilde{R}_g\Big(\alpha_g\psi - \beta_g\ ,\ \alpha_{-g}\theta - \beta_{-g}\ ,\ \theta\ ,\ \psi\Big) > 0$ and $\left(\alpha_{-g}\partial_2 + \partial_3\right)\tilde{R}_g\Big(\alpha_g\psi - \beta_g\ ,\ \alpha_{-g}\theta -$

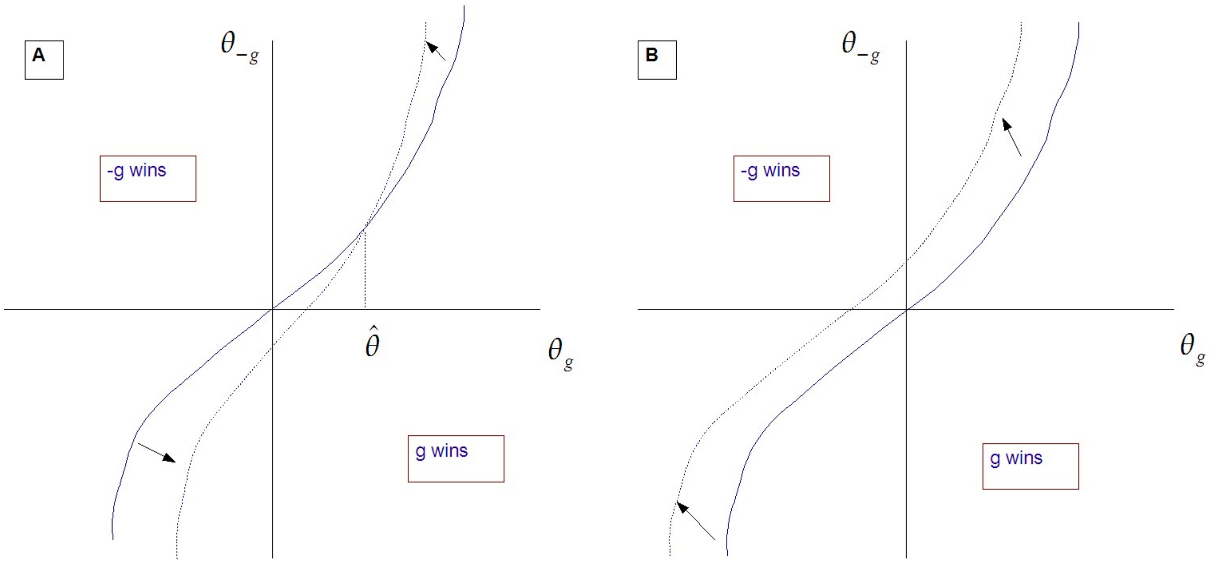

Figure 2: **Comparative Statics.** **A**: effect of an increase in p_g on the *frontier function* ψ_g when the parameters lie in $\mathcal{U}_-$. **B**: effect of a decrease in γ_g on the *frontier function* ψ_g when the parameters lie in $\mathcal{U}_-$.

$\beta_{-g}\ ,\ \theta\ ,\ \psi\Big) < 0,$

or

(ii) $(\alpha_g\partial_1+\partial_4)\tilde{R}_g\Big(\alpha_g\psi-\beta_g\ ,\ \alpha_{-g}\theta-\beta_{-g}\ ,\ \theta\ ,\ \psi\Big) < 0$ and $(\alpha_{-g}\partial_2+\partial_3)\tilde{R}_g\Big(\alpha_g\psi-\beta_g\ ,\ \alpha_{-g}\theta-\beta_{-g}\ ,\ \theta\ ,\ \psi\Big) > 0,$

Let $\mathcal{U}_-$ denote the reunion of all the open sets of parameters that fall in the first category, and $\mathcal{U}_+$ denote the union of open sets of parameters that fall in the second category. The following proposition gives a summary of all the comparative statics results. Intuitions and comments follow the proposition.

Proposition 3 (Comparative Statics)**.**

1. *On* $\mathcal{U}_-$*:*

   (i) ψ_g *is increasing (decreasing) in* p_g*,* $-P_g$*,* $-p_{-g}$ *and* P_{-g} *for high (low)* θ_g*s.*

   (ii) ψ_g *is increasing in* y_g*,* γ_g*,* $-y_{-g}$ *and* $-\gamma_{-g}$ *for every* θ_g*.*

2. *On* $\mathcal{U}_+$*:*

(i) ψ_g is increasing (decreasing) in $-p_g$, P_g, p_{-g} and $-P_{-g}$ for high (low) θ_gs.

(ii) ψ_g is increasing in $-y_g$, $-\gamma_g$, y_{-g} and γ_{-g} for every θ_g.

Proof. See Appendix A. □

The claim of the proposition was made a bit imprecise in order to make a global statement. The precise meaning of the proposition is best illustrated by an example. Fix a vector of parameters $(p_g, P_g, y_g, \gamma_g)_{g\in\{-1,1\}}$, suppose this vector lies in $\mathcal{U}_-$, and consider the change of the frontier function $\psi_g(.)$ induced by a marginal increase in the precision of the private information p_g of group g. Then, there exists a $\hat{\theta}$ in $\mathbb{R}$ such that all the points of the graph of the function $\psi_g(.)$ such that $\theta_g > \hat{\theta}$ move upward, while all the points such that $\theta_g < \hat{\theta}$ move downward (see Figure 2). $\hat{\theta}$ is the point of the frontier at which the comparative statics with respect to p_g switches. The proposition says that such a switching point can be found for each of the precision parameters, but it is generally not the same for, say, p_g and P_g, and it also varies with the point considered in the parameter space (the vector $(p_g, P_g, y_g, \gamma_g)_{g\in\{-1,1\}}$ in our example). For realized values of the public signal y_g, y_{-g} and payoff terms γ_g and γ_{-g}, the proposition says that they induce uniform shifts of the winning sets, and that the direction of these shifts depends on whether the parameters lie in $\mathcal{U}_-$ or $\mathcal{U}_+$. A few remarks help making a general sense of this comparative statics exercise that uncovers several interesting insights.

Remark 1. *No marginal change of the information structure can uniformly expand the winning set of a given group in the space of parameter strengths.*

Remark 2. *Changes in the payoff structure or in the realizations of the public signals induce uniform shifts of the winning sets of the groups.*

More precisely, any change in one parameter of the information structure that gives the edge to one of two sufficiently strong tied groups would have the opposite effect for two sufficiently weak tied groups. On the other hand, a change in the payoff structure or in the realized public signals always strictly expands the winning set of one of the groups.

Remark 3. *Private and public precisions of a given group always have opposite effects on the frontier.*

As it is intuitive, public information favors collective action by facilitating coordination while private information makes agents beliefs less correlated. This intuition is confirmed in Proposition 4.

Remark 4. *While the public signals and the payoff parameters have natural effects on the frontier in $\mathcal{U}_-$, their effects are reversed in $\mathcal{U}_+$.*

This remark provides an interesting result. Indeed, remember that from Proposition 2, when public precisions are sufficiently precise relative to private precisions, the parameters must lie in $\mathcal{U}_+$. The comparative statics implies that when public information is precise in this way, bad news become good news, and bad incentives become good incentives. The intuition is that when public information is relatively precise, bad news or bad incentives make all the agents more aggressive about their private information. Therefore, if the first remarks seem to suggest a limited role for the information structure, or at least an ambiguous one, the latter remark suggests a much more important role: the relative precisions of private and public information can radically change the role of incentives and public signals. For example, I mentioned above that the incentive parameter γ was decreasing in the ability of the group to identify and punish free-riders in case of victory. The latter remark implies that when public information is relatively precise it is better for the group to be softer on free-riders or to detect them inefficiently. In the same way, an enemy who threatens to punish those active members of the other group in case of defeat may obtain a result opposite to the one intended if public information is relatively precise.

The last result of this section illustrates the links between public information and collective action. It shows that increasing marginally the sensitivity of the *social rule* to the collective action (*activity rate*) of group g has the same effect as increasing marginally the precision of public information for that group, or decreasing marginally the precision of private information for that group. I formalize the increase in sensitivity as follows. Given a social rule R_g, I consider the social rules R_g^λ for $\lambda > 0$ which are defined by

$$\tilde{R}_g^\lambda(l_g, l_{-g}, \theta_g, \theta_{-g}) = \tilde{R}_g(\lambda l_g, l_{-g}, \theta_g, \theta_{-g}).$$

Social rules with a higher λ are more sensitive to the *activity rate* of group g. I let ψ_g^λ denote the *frontier function* associated with the social rule R_g^λ. It solves the equation

$$\tilde{R}_g\Big(\lambda(\alpha_g\psi - \beta_g)\ ,\ \alpha_{-g}\theta - \beta_{-g}\ ,\ \theta\ ,\ \psi\Big) = 0. \tag{18}$$

Proposition 4.

1. *On $\mathcal{U}_-$, ψ_g^λ is decreasing in λ for high θ_gs and increasing in λ for low θ_gs.*

2. *On $\mathcal{U}_+$, ψ_g^λ is increasing in λ for high θ_gs and decreasing in λ for low θ_gs.*

Proof. See Appendix A. □

5.2 A Particular Case

In the remainder of the paper, the social rule is that of Example 2, which compares scoring functions given by

$$s(l_g, \theta_g) = \Phi^{-1}\big(l_g\big) + \theta_g. \tag{19}$$

This defines a social rule that satisfies all the required assumptions but does not satisfy (DR). It has the advantage that all the partial derivatives of the corresponding function $\tilde{R}_g$ are constant. It is easy to prove that for this case the unique equilibrium of the game that satisfies (SMSCP) is defined by the frontier function

$$\psi_g(\theta_g) = \frac{\alpha_{-g} - 1}{\alpha_g - 1}\theta_g + \frac{\beta_g - \beta_{-g}}{\alpha_g - 1}. \tag{20}$$

It is strictly increasing (and hence indeed a *frontier function*) on the set $\mathcal{U} = \mathcal{U}_- \cup \mathcal{U}_+$ where $\mathcal{U}_-$ and $\mathcal{U}_+$ correspond to their definition in the general case and are here defined precisely by

$$\mathcal{U}_- = \left\{ (p_g, p_{-g}, P_g, P_{-g}) \in \mathbb{R}^4 | \forall g,\ 1 < \alpha_g \right\} \tag{21}$$

$$\mathcal{U}_+ = \left\{ (p_g, p_{-g}, P_g, P_{-g}) \in \mathbb{R}^4 | \forall g,\ 1 > \alpha_g \right\}. \tag{22}$$

Hence $\mathcal{U}_-$ is the region of low relative private precisions while $\mathcal{U}_+$ is the region of high relative private precisions. The comparative statics of Section 5.1 naturally applies here. The advantage of this example is that it makes the calculation of the *ex ante* probability of winning tractable, and thus enables *ex ante* comparative statics with respect to the information structure. In order to make the expressions less cumbersome, I state the result assuming symmetric cost parameters $\gamma_{-1} = \gamma_1 = \gamma$ and precision of private information $p_g = p_{-g} = p$ and perform comparative statics with respect to the public precisions. This is done in the spirit of the next section that considers the public precisions as a policy variable for different decision makers. Public precision is arguably a more reasonable policy variable than private precision.

Proposition 5.

1. *If costs are low ($\gamma < 1/2$) and the precision of public information is relatively high in both teams ($P_g, P_{-g} > \sqrt{p} - \bar{p}$), then $Pr[g$ wins$] > 1/2$ if and only if $P_{-g} > P_g$. The same holds if costs are high ($\gamma > 1/2$) and the precision of public information is low ($P_g, P_{-g} < \sqrt{p} - \bar{P}$).*

2. *If costs are low ($\gamma < 1/2$) and the precision of public information is relatively low in both teams($P_g, P_{-g} > \sqrt{p} - \bar{p}$), then $Pr[g$ wins$] > 1/2$ if and only if $P_g > P_{-g}$. The same holds if costs are high ($\gamma > 1/2$) and the precision of public information is high ($P_g, P_{-g} < \sqrt{p} - \bar{P}$).*

Proof. See Appendix A. □

6 Endogenous Information Structure

6.1 A Strategic Game of Disclosure

Suppose each team is managed by a *team leader* whose objective is to maximize the probability that her team wins the contest. The only information available to the *team leaders* is the prior on both θ_g, θ_{-g}, in particular they don't know their own team's *strength*. Based on this information, they commit to a disclosure policy, that is they pick the precision of the noisy public signal that they will send to the other team. That is team g chooses P_{-g}. The teams are assumed to be symmetric in every other respect, that is in costs and precision of the private information. Their choice is based on the assumption that in the subsequent game played by the agents of both teams the equilibrium in strategy profiles that satisfy (SMSCP), described above, will be reached. This is problematic because for this example because, as mentioned above, there are multiple equilibria since full activity on one side and full inactivity on the other side is an equilibrium. The use of equilibria that satisfy (SMSCP) also implies limitations in the strategy spaces available to the players, since for some pairs of actions (P_g, P_{-g}) a *responsive equilibrium* does not exist. Let $\mathcal{K} = [\underline{K}, \overline{K}]^2 \subset \mathbb{R}^2_{++}$ be a compact square from which team leaders pick their actions (P_g, P_{-g}). If $\mathcal{K} \subset \mathcal{U}_+$ or $\mathcal{K} \subset \mathcal{U}_-$, the problem of non-existence of a *responsive equilibrium* does not bind.

Proposition 6.

1. *If $\gamma < 1/2$, and $\mathcal{K} \subset \mathcal{U}_+$, then the unique Nash equilibrium of the disclosure game is $P_g = P_{-g} = \overline{K}$.*

2. *If $\gamma < 1/2$, and $\mathcal{K} \subset \mathcal{U}_-$, then the unique Nash equilibrium of the disclosure game is $P_g = P_{-g} = \underline{K}$.*

3. *If $\gamma > 1/2$, and $\mathcal{K} \subset \mathcal{U}_+$, then the unique Nash equilibrium of the disclosure game is $P_g = P_{-g} = \underline{K}$.*

4. If $\gamma > 1/2$, and $\mathcal{K} \subset \mathcal{U}_-$, then the unique Nash equilibrium of the disclosure game is $P_g = P_{-g} = \overline{K}$.

Proof. See Appendix A. □

If instead I assumed that the team leaders are playing a game where they decide *ex ante* (with commitment) on the level of noise that they will use to transmit public information about the strength of the other team that they learn ex interim to their own team, the results above would just need to be reversed. That is, the results that obtain for low costs above would obtain for high costs in this new game, and *vice versa*.

6.2 A Contest Design Problem

Knowing how the game is played, given a certain information structure, allows a *social planner* or a *contest designer* to design appropriate rules about information disclosure. Assume that the designer controls the precision of the public information exchanged between the two teams. There are externalities that make the efforts expended by both teams valuable, so that the designer wants to maximize the expected total participation in the contest. However she also takes into account the money transfer needed to reward the winners and the utilities of all the contestants. The players receive a monetary payoff of 1 from the designer when they are active in the winning team, 0 if they are not active, and pay an effort cost γ whenever they are active. The objective function of the designer is then given by

$$\alpha(l_g + l_{-g}) - l^{\text{win}} + \left(l^{\text{win}} - \gamma l^{\text{win}}\right) - \gamma l^{\text{lose}} = (\alpha - \gamma)(l_g + l_{-g}) \tag{23}$$

where l^{win} and l^{lose} are the participation rates in the winning team and the losing team and α is how much she valued total participation. I assume $\alpha > \gamma$.

The designer is therefore maximizing the expected total participation given her information *ex ante*. Assume that the two teams have symmetric cost structures and private information precisions, and that the designer is constrained to choose a symmetric rule for public information disclosure. In this case, by symmetry, the designer maximizes

$$E\left(l_g + l_{-g}\right) = 2E(l_g). \tag{24}$$

These assumptions lead to the following result

Proposition 7. *If costs to participate are low ($\gamma < 1/2$), a full disclosure policy $P \to \infty$ is optimal for the designer, and the maximal expected total participation is then $2(1-\gamma)$. If costs to participate are high ($\gamma > 1/2$) then a partial disclosure policy with $P = \sqrt{p} - \bar{P}$ is optimal and the maximal expected total participation is 1. In the knife-edge case where $\gamma = 1/2$, the choice of disclosure rule is irrelevant.*

Proof. See Appendix A □

In particular, the optimal participation rate is linearly decreasing in the cost parameter γ for low costs, but stays constant after γ reaches $1/2$. Also, it is independent from the private information structure (even though the private information structure does affect the optimal choice of the public information structure for high costs).

7 Applications

This section provides some examples of situations that are reasonably well described by the model

Political Competition between fragmented parties. Two political parties compete for power. Each of the parties is infinitely fragmented in the sense that it is composed of many politicians or factions with a small individual weight who take independent decisions about whether to support their party in the campaign. In case of success, support is rewarding since it gives access to political decision. However, it entails a cost since it associates the individual politician or the faction to the official positions of the party during this particular campaign, potentially jeopardizing their future careers. Hence it pays to join the party in case of success, but it is better to abstain in case of failure. The chances of success depend on the number of participants in each party, and on some other idiosyncratic characteristics of the parties. To capture that, each party has a scoring function, and the political process is assumed to accurately appoint the party with the highest score. The idiosyncratic characteristic of a party could be for example the quality of its program, the degree of mobilization among its partisans, the quality of its communication strategy, the quantity of funds available for campaign financing or any other attribute that is relevant for success, and about which it could be reasonable to assume asymmetric information. Indeed, each politician is assumed to know the characteristic of its own party perfectly, while being only imperfectly informed about the characteristics of the other party.

Contribution Races. Two projects compete for implementation by a central authority. The performance of each project depends on an inherent characteristic and on its ability to raise contributions among its partisans. An example could be lobbying. Two special-interest groups support incompatible policies. The inherent characteristic of a policy is its marketability towards a wide audience. In order to prevail, each group needs to be able to raise funds among its supporters. Another example is one of two competing projects each with their pool of potential investors (venture capitalists) who decide whether to back the project and invest in it. One can also think about two competing theories in a research field, each endowed with a pool of researchers who could possibly contribute a paper to a theory.

Ethnic Conflicts. In an ethnically divided society, the members of an ethnic group benefit if power is seized by one of their members. However, in a transition phase, they may also be reluctant to support a leader from their group in case another group prevails and decides to punish the supporters of his opponents. The beliefs of the agents in each group will matter crucially in this case, and the leaders may decide to engage in demonstrations of force to convey a message to other ethnic groups and try to deter them from supporting their own leaders.

Team Bonus within a Firm. Firms often try to give incentives to its employees by awarding a bonus to the team that performs best. To fix ideas, think of a Fund with teams of traders that operate in different markets, or a retail group with teams of sellers in different locations. The performance of each team depends on the total effort of its members, and on some exogenous parameter (the strength parameter) that is idiosyncratic to each team. This parameter can be a characteristic of the environment that each team is operating in, or a characteristic of the technology operated by the team (where technology is understood broadly): for instance characteristics of the specific market in which a team is operating for the Fund, and local demands for the retail group. The members of each team decide whether to exercise effort. They know their team parameter perfectly but have imperfect and idiosyncratic private information and public information about the opponents' team parameter. A rationale for the idiosyncratic component may be that employees have different domains of expertise.

Wars. In wars, information about the opponent is crucial not only to the generals but also to soldiers who may behave very differently depending on their beliefs about the forces of the enemy. There is indeed a coordination problem, and for an army to be strong, its members need to be confident about their chances but also about the beliefs of the rest of the army. The importance of morale has been long recognized, and the way the use of the media has been scrutinized and questioned in wars of the past century, leading to very different responses from the Vietnam war (complete transparency) to the first Gulf war (opacity) may be an illustration of the importance of the link between information and beliefs of the agents. Other issues can be discussed with the framework proposed here. For example, a possible interpretation of the surprise effect is that it creates lack of common knowledge among a group. The private signals received by the agents are very dispersed and the lack of time to communicate, even by eye contact, does not allow agents of the group to improve the precision of their private signal by sampling the signals of other agents. Hence surprise affects the information structure within the group, and can lead to lack of coordination.

8 Conclusion

This paper analyzes the role of the information that a continuum of agents hold about their opponents in a model of conflict between two groups, and proposes an original way to look at a strategic interaction both within and across groups. It suggests an approach to think about the role of public and private information in social competition across groups. The second part of the paper links the decision of a leader to inform her followers more or less precisely to their incentives to contribute.

A Proofs

Proof of Lemma 1. If the strategies are *strictly monotonic*, then it is easy to see that $l_g(\theta_g, \theta_{-g})$ has to be *strictly monotonic*, and therefore so must be the bias induced by the strategy profile $R^\pi(\theta_g, \theta_{-g})$. □

Proof of Proposition 1. The probability of winning given the frontier function $\psi_g(.)$ is strictly decreasing in the best forecast of the agent $m_g(x, y)$. Therefore an agent is better off being active if and only if $m_g(x, y)$ is below a threshold $\hat{m}_g$, where $\hat{m}_g$ is the threshold that equalizes her probability of winning (6) to her *cost of activity* γ_g, leading to (3) and the optimal strategies of the proposition.

By the law of large numbers, the activity rate of group g is equal to the probability that an agent of group g has a best forecast below $\hat{m}_g$, or equivalently a private signal x below

$$\hat{x} = \frac{\Pi_g}{p_g} \left(\hat{m}_g - \frac{P_g}{\Pi_g} y_g \right).$$

And using the definition of the signalling technology in (1), this corresponds to a realized noise in the private signal equal to $\hat{\varepsilon_g}$ as defined in (9) of the proposition. The activity rate in (8) obtains from the fact that this noise follows a standard normal distribution. □

Proof of Theorem 1. On the frontier, $\theta_{-g} = \psi_g(\theta_g)$. Replacing this in (9) yields the following activity rate for group g on the frontier $l_g^f = \Phi\big(\alpha_g \psi_g(\theta_g) - \beta_g\big)$ where the expressions for α_g and β_g are given in (11) and (12). By symmetry, $l_{-g}^f = \Phi\big(\alpha_{-g}\psi_{-g}(\theta_{-g}) - \beta_{-g}\big)$, but since $\psi_{-g} = \big(\psi_g\big)^{-1}$, I can write $l_{-g}^f = \Phi\big(\alpha_{-g}\theta_g - \beta_{-g}\big)$. Therefore the bias function on the frontier is given by $\tilde{R}_g\Big(\alpha_g \psi - \beta_g \ , \ \alpha_{-g}\theta - \beta_{-g} \ , \ \theta \ , \ \psi\Big)$. But since no team wins on the frontier, this bias must be equal to 0, yielding to the characterization of the frontier in (13). Hence, if an equilibrium that satisfies (SMSCP) exists, it must satisfy condition (ii) of the proposition by definition, and, as I just showed, it must also satisfy (i). Suppose instead that there is a function that satisfies these conditions, letting $\psi_g = \psi$ and $\psi_{-g} = \psi^{-1}$ and using the strategies defined by Proposition 1 provides an equilibrium that satisfies SMSCP. $\square$

Proof of Theorem 2. Let $f(\theta, \psi) = \tilde{R}_g\Big(\alpha_g \psi - \beta_g \ , \ \alpha_{-g}\theta - \beta_{-g} \ , \ \theta \ , \ \psi\Big)$. Following Lemma 2, a sufficient condition for the existence of a solution to the implicit equation $f(\theta, \psi) = 0$ is that there exists $d > 0$ such that for every $(\theta, \psi) \in \mathbb{R}^2$

$$\left|\frac{\partial}{\partial \psi} f(\theta, \psi)\right| = \left|\Big(\alpha_g \partial_1 + \partial_4\Big)\tilde{R}_g\Big(\alpha_g \psi - \beta_g \ , \ \alpha_{-g}\theta - \beta_{-g} \ , \ \theta \ , \ \psi\Big)\right| \geq d. \tag{25}$$

However, I also need the solution to be strictly increasing for our purpose. But since the derivative of a solution ψ is given by (16), it is easy to see that the existence of a $d > 0$ such that (15) holds for every $(\theta, \psi) \in \mathbb{R}^2$ ensures both that the sufficient condition for existence is satisfied and that the solution is strictly increasing. Indeed, because $\tilde{R}_g$ is continuously differentiable, the function $\Big(\alpha_g \partial_1 + \partial_4\Big)\tilde{R}_g\Big(\alpha_g \psi - \beta_g \ , \ \alpha_{-g}\theta - \beta_{-g} \ , \ \theta \ , \ \psi\Big)$ cannot change sign while still satisfying (25) for every (θ, ψ). $\square$

Proof of Theorem 3. From the definition of (DR), I define two functions $\overline{\psi}\,^0_1 = \overline{\psi}$ and $\underline{\psi}\,^0_1 = \underline{\psi}$, where the index stands for the group. Since they are strictly increasing by assumption, I can define their inverses $\overline{\psi}\,^0_{-1} = \left[\underline{\psi}\,^0_1\right]^{-1}$ and $\underline{\psi}\,^0_{-1} = \left[\overline{\psi}\,^0_1\right]^{-1}$. With the (DR) assumption, a member of group g who believes with sufficiently high probability that θ_{-g} is above $\overline{\psi}\,^0_g(\theta_g)$ will decide to remain passive, while if she believes with sufficiently high probability that θ_{-g} is below $\underline{\psi}\,^0_g(\theta_g)$

she will decide to become active. This yields some bounds on the *activity rates* l_g and l_{-g} which in turn yield new frontiers $\underline{\psi}^{\,1}_{g}$ and $\overline{\psi}^{\,1}_{g}$. Iterating this process leads to functions that bound any possible equilibrium frontier function above and below. I construct these functions and show that under the assumptions of the theorem they must be equal to the frontier function of the unique equilibrium that satisfies (SMSCP).

Let $\overline{\psi}^{\,n}_{g}$ and $\underline{\psi}^{\,n}_{g}$ be two strictly increasing continuously differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ such that g wins whenever $\theta_{-g} < \underline{\psi}^{\,n}_{g}(\theta_g)$ and loses whenever $\theta_{-g} > \overline{\psi}^{\,n}_{g}(\theta_g)$. Then any agent in g who believes with a probability higher than γ_g that $\theta_{-g} < \underline{\psi}^{\,n}_{g}$ chooses to be active. The number of such agents is given by

$$\underline{l}^{\,n}_{g} = \Phi\left(p_g^{-1/2}\big(\Pi_g \underline{\psi}^{\,n}_{g}(\theta_g) - p_g\theta_{-g} - \Pi_g^{1/2}\Phi_{-1}(\gamma_g) - P_g y_g\big)\right).$$

Similarly, I can calculate the proportion of agents who believe that $\theta_{-g} > \overline{\psi}^{\,n}_{g}(\theta_g)$ with a probability higher than $1-\gamma_g$, and hence remain passive for sure, and obtain the following upper bound on the number of agents who become active in group g

$$\overline{l}^{\,n}_{g} = \Phi\left(p_g^{-1/2}\big(\Pi_g \overline{\psi}^{\,n}_{g}(\theta_g) - p_g\theta_{-g} - \Pi_g^{1/2}\Phi_{-1}(\gamma_g) - P_g y_g\big)\right).$$

Having done that for both groups, I can obtain bounds on the *ex post* bias for group g

$$\underline{R}^{\,n}_{g}(\theta_g,\theta_{-g}) = R_g\big(\underline{l}^{\,n}_{g}, \overline{l}^{\,n}_{-g}, \theta_g, \theta_{-g}\big) \leq R_g \leq R_g\big(\overline{l}^{\,n}_{g}, \underline{l}^{\,n}_{-g}, \theta_g, \theta_{-g}\big) = \overline{R}^{\,n}_{g}(\theta_g,\theta_{-g}).$$

For each θ_g, I can find the value of θ_{-g} that would make the upper bound $\overline{R}^{\,n}_{g}(\theta_g,\theta_{-g})$ equal to 0. Let $\overline{\psi}^{\,n+1}_{g}(\theta_g)$ this value of θ_{-g}. Then $\overline{\psi}^{\,n+1}_{g}(\theta_g)$ must solve the equation

$$\underbrace{\tilde{R}_g\left(p_g^{-1/2}\Pi_g\overline{\psi}^{\,n}_{g}(\theta_g) - p_g^{1/2}\overline{\psi}^{\,n+1}_{g}(\theta_g) - \beta_g, p_{-g}^{1/2}\Pi_{-g}\left[\overline{\psi}^{\,n}_{g}\right]^{-1}(\overline{\psi}^{\,n+1}_{g}) - p_{-g}^{1/2}\theta_g - \beta_{-g}, \theta_g, \overline{\psi}^{\,n+1}_{g}\right)}_{\equiv f^n\left(\theta_g, \overline{\psi}^{\,n+1}_{g}\right)} = 0. \tag{26}$$

It is easy to see that f^n is continuously differentiable in both arguments, strictly increasing in its first argument and strictly decreasing in the second one. It implies that $\overline{\psi}_g^{n+1}$ exists, is uniquely defined by this equation, and it is continuously differentiable and strictly increasing. Furthermore, because $-g$ wins whenever $\theta_{-g} > \overline{\psi}_g^n(\theta_g)$, it must be true that $\overline{\psi}_g^{n+1}(\theta_g) \leq \overline{\psi}_g^n(\theta_g)$. I can define $\underline{\psi}_g^{n+1}$ in the same way. And it is true that g wins whenever $\theta_{-g} < \underline{\psi}_g^{n+1}(\theta_g)$ and loses whenever $\theta_{-g} > \overline{\psi}_g^{n+1}(\theta_g)$. In particular this implies that for every θ_g $\underline{\psi}_g^{n+1}(\theta_g) \leq \overline{\psi}_g^{n+1}(\theta_g)$.

Initiating from the frontiers of the dominance regions $\underline{\psi}_g^0(.)$, and $\overline{\psi}_g^0(.)$, I have obtained two sequences of continuously differentiable and strictly increasing functions from $\mathbb{R}$ to $\mathbb{R}$, and the sequence $\{\underline{\psi}_g^n\}$ is increasing pointwise while the sequence $\{\overline{\psi}_g^n\}$ is decreasing pointwise. Because for every θ_g and every n $\underline{\psi}_g^n(\theta_g) \leq \overline{\psi}_g^n(\theta_g)$ each of these sequences converges weakly. I denote the pointwise limits of these sequences by $\underline{\psi}_g^\infty(\theta_g)$ and $\overline{\psi}_g^\infty(\theta_g)$ for each θ_g. The functions $\underline{\psi}_g^\infty(.)$ and $\overline{\psi}_g^\infty(.)$ must be weakly increasing by standard arguments, and satisfy $\underline{\psi}_g^\infty(\theta_g) \leq \overline{\psi}_g^\infty(\theta_g)$ for every θ_g.

I finish the proof by showing that, when there exists a unique equilibrium that satisfies (SMSCP), characterized by a frontier function ψ_g, then the sequences above must converge to that function, $\underline{\psi}_g^\infty = \psi_g = \overline{\psi}_g^\infty$.

For this, fix some θ_g, and consider the sequences defined by $u_n = \overline{\psi}_g^n(\theta_g)$ and $v_n = \left[\overline{\psi}_g^n\right]^{-1}(u_{n+1})$. I already know that $\{u_n\} \to \overline{\psi}_g^\infty(\theta_g)$. And it is also easy to see that if $\overline{\psi}_g^\infty$ is continuous and strictly increasing on an open neighborhood of θ_g, then $\{v_n\} \to \theta_g$. It is more problematic to characterize the behavior of this sequence when θ_g is a discontinuity point, or when θ_g belongs to a closed interval on which $\overline{\psi}_g^\infty$ is constant. I know that, because $\overline{\psi}_g^\infty$ is weakly increasing, there are countably many such discontinuity points and constant valued intervals. I can show that when θ_g is a discontinuity point for $\overline{\psi}_g^\infty$ or the left extremity of a constant-valued interval, then it is also true that $\{v_n\} \to \theta_g$. Hence at any of these points, taking the limits in (26), and by continuity of $\hat{R}_g$, I obtain that

$$\tilde{R}_g\left(p_g^{-1/2}\Pi_g\overline{\psi}_g^{\infty}(\theta_g) - p_g^{1/2}\overline{\psi}_g^{\infty}(\theta_g) - \beta_g, p_{-g}^{1/2}\Pi_{-g}\theta_g - p_{-g}^{1/2}\theta_g - \beta_{-g}, \theta_g, \overline{\psi}_g^{\infty}\right) = 0. \tag{27}$$

That is, after simplification, $\overline{\psi}_g^{\infty}(\theta_g)$ must solve (13). But the only solution to that equation is given by the frontier function ψ_g by assumption. Therefore, $\overline{\psi}_g^{\infty}$ must coincide with ψ_g on every open set on which it is strictly increasing, at every discontinuity point and at every left end of a constant valued interval. This implies in particular that at every θ_g, $\overline{\psi}_g^{\infty}(\theta_g) \leq \psi_g(\theta_g)$. Indeed, suppose that at some θ, $\overline{\psi}_g^{\infty}(\theta) > \psi_g(\theta)$. Then there must exist an open interval and neighborhood of θ $\mathcal{V}$ on which $\overline{\psi}_g^{\infty}$ is constant valued. Consider the longest possible interval containing θ and on which $\overline{\psi}_g^{\infty}$ is constant valued. It must be of finite size because otherwise it would reach the *dominated regions* which is impossible. And because $\overline{\psi}_g^{\infty}$ is left-continuous, it reaches its left boundary θ^-. Hence $\overline{\psi}_g^{\infty}(\theta^-) = \overline{\psi}_g^{\infty}(\theta) > \psi_g(\theta) > \psi_g(\theta_-)$. But θ^- must be either a discontinuity point of $\overline{\psi}_g^{\infty}$, or there must be an interval to the left of θ_- where $\overline{\psi}_g^{\infty}$ is strictly increasing. In the first case, because of what I found about discontinuity points, and in the second case, because of what I found about open intervals where $\overline{\psi}_g^{\infty}$ is strictly increasing and by continuity, I must have $\overline{\psi}_g^{\infty}(\theta_-) = \psi_g(\theta_-)$, which is a contradiction.

The same reasoning for $\underline{\psi}_g^{\infty}(\theta_g)$ leads to the conclusion that for every θ_g, $\underline{\psi}_g^{\infty}(\theta_g) \geq \psi_g(\theta_g)$. But since for every θ_g, I know that $\underline{\psi}_g^{\infty}(\theta_g) \leq \overline{\psi}_g^{\infty}(\theta_g)$, it must be true that, at every θ_g, $\underline{\psi}_g^{\infty}(\theta_g) = \overline{\psi}_g^{\infty}(\theta_g)\psi_g(\theta_g)$. This concludes the main part of the proof.

To see that the equilibrium strategy profile is the only one that survives iterated elimination of strictly dominated strategies, notice that each new frontier function is obtained from the former one by eliminating strictly dominated strategies for all the players. □

Proof of Proposition 2. (i) In this case, and considering that $\partial_1\tilde{R}_g, \partial_3\tilde{R}_g > 0$ and $\partial_2\tilde{R}_g, \partial_4\tilde{R}_g < 0$, I have for every $z \in \mathbb{R}^4$

$$\left(\alpha_g\partial_1 + \partial_4\right)\tilde{R}_g(z) \geq \left(\frac{1}{m}\partial_1 + \partial_4\right)\tilde{R}_g(z) \geq \eta,$$

and

$$-\big(\alpha_{-g}\partial_2+\partial_3\big)\tilde{R}_g(z)\geq-\left(\frac{1}{m}\partial_2+\partial_3\right)\tilde{R}_g(z)\geq\eta,$$

so that the sufficient condition of Theorem 2 is satisfied.

(ii) In this case, I have for every $z\in\mathbb{R}^4$

$$-\big(\alpha_g\partial_1+\partial_4\big)\tilde{R}_g(z)\geq-\left(\frac{1}{M}\partial_1+\partial_4\right)\tilde{R}_g(z)\geq\eta,$$

and,

$$\big(\alpha_g\partial_2+\partial_3\big)\tilde{R}_g(z)\geq\left(\frac{1}{M}\partial_2+\partial_3\right)\tilde{R}_g(z)\geq\eta,$$

and the sufficient condition of Theorem 2 is again satisfied.

□

Proof of Proposition 3. I show the comparative statics results for p_g and y_g. Other results are obtained similarly. Let $u=\big(p_g,P_g,y_g,\gamma_g\big)_{g\in\{-1,1\}}$ be a vector of parameters such that a *frontier function* $\psi_g(.,u)$ is defined in an open neighborhood $\mathcal{V}$ of u. Then an application of Lemma 2 shows that this frontier function $\psi_g(.,u)$ is continuously differentiable in the parameters on $\mathcal{V}$, and its derivative with respect to any of the parameters can be obtained by implicit differentiation of (13). To simplify notation I let $v=\big(\alpha_g\psi_g(\theta_g)-\beta_g,\alpha_{-g}\theta_g-\beta_{-g},\theta_g,\psi_g(\theta_g)\big)$. Differentiating (13) with respect to p_g yields

$$\frac{\partial}{\partial p_g}\psi_g(\theta_g,u)=-\underbrace{\left(\frac{\partial\alpha_g}{\partial p_g}\psi_g(\theta_g)-\frac{\partial\beta_g}{\partial p_g}\right)}_{L}\underbrace{\left(\frac{\partial_1}{\alpha_g\partial_1+\partial_4}\right)\tilde{R}_g(v)}_{R}. \tag{28}$$

Since ψ_g is strictly increasing in θ_g and $\partial\alpha_g/\partial p_g<0$, there exists $\hat{\theta}$ such that $L>0$ for $\theta_g<\hat{\theta}$ and $L<0$ for $\theta_g>\hat{\theta}$. The other term R is positive on $\mathcal{U}_-$ and negative $\mathcal{U}_+$. Therefore, on $\mathcal{U}_-$, ψ_g is decreasing in p_g for $\theta_g<\hat{\theta}$ and increasing in p_g for $\theta_g>\hat{\theta}$, and the opposite is true on $\mathcal{U}_+$.

Differentiating with respect to y_g yields

$$\frac{\partial}{\partial p_g}\psi_g(\theta_g,u)=\frac{\partial\beta_g}{\partial y_g}\underbrace{\left(\frac{\partial_1}{\alpha_g\partial_1+\partial_4}\right)\tilde{R}_g(v)}_{R}. \tag{29}$$

$\partial\beta_g/\partial y_g>0$ and the second term is the same as before. Hence, ψ_g is increasing in y_g on $\mathcal{U}_-$ and decreasing in y_g on $\mathcal{U}_+$. □

Proof of Proposition 4. The result is obtained by differentiating (18) with respect to λ. Let $v=\big(\lambda(\alpha_g\psi_g(\theta_g)-\beta_g),\alpha_{-g}\theta_g-\beta_{-g},\theta_g,\psi_g(\theta_g)\big)$ and let $u=\big(p_g,P_g,y_g,\gamma_g\big)_{g\in\{-1,1\}}$ be the the parameter vector. I obtain

$$\frac{\partial}{\partial\lambda}\psi_g^{\lambda}(\theta_g)=-\big(\alpha_g\psi_g^{\lambda}(\theta_g)-\beta_g\big)\left(\frac{\partial_1}{\lambda\alpha_g\partial_1+\partial_4}\right)\tilde{R}_g(v). \tag{30}$$

If $u\in\mathcal{U}_-$, there exists $\hat{\theta}$ such that this expression is negative for $\theta_g<\hat{\theta}$ and positive for $\theta_g>\hat{\theta}$, while if $u\in\mathcal{U}_+$, there exists $\hat{\theta}$ such that this expression is positive for $\theta_g<\hat{\theta}$ and negative for $\theta_g>\hat{\theta}$. □

Proof of Proposition 5. I start by calculating the probability of winning of a group at the *ex ante* stage. For g, it is the probability that $\theta_{-g}<\psi_g(\theta_g)$ measured with the prior distribution on (θ_g,θ_{-g}). The event can be written

$$|\alpha_g-1|\theta_g<|\alpha_{-g}-1|\theta_{-g}+\Lambda_{\mathcal{U}}(\beta_{-g}-\beta_g),$$

where the indicator function $\Lambda_{\mathcal{U}}=\mathbf{1}_{\mathcal{U}_+}-\mathbf{1}_{\mathcal{U}_-}$ is equal to 1 when the precisions lie on $\mathcal{U}_+$ and to -1 when the precisions lie on $\mathcal{U}_-$. This can be rewritten as

$$\begin{aligned}\Lambda_{\mathcal{U}}\left((1-p_g^{-1/2}\bar{P})\theta_g-(1-p_{-g}^{-1/2}\bar{P})\theta_{-g}+\sqrt{\frac{P_{-g}}{p_{-g}}}\eta_{-g}-\sqrt{\frac{P_g}{p_g}}\eta_g\right)&\\<\Lambda_{\mathcal{U}}\left(\sqrt{\frac{\Pi_g}{p_g}}\Phi^{-1}(\gamma_g)-\sqrt{\frac{\Pi_{-g}}{p_{-g}}}\Phi^{-1}(\gamma_{-g})\right).&\end{aligned} \tag{31}$$

According to the prior, the left hand-side of this equation is normally distributed with variance

$$\sigma^2 = \frac{P_g + \bar{P}}{p_g} + \frac{P_{-g} + \bar{P}}{p_{-g}} + 2\left(\bar{P}^{-1} - p_g^{-1/2} - p_{-g}^{-1/2}\right), \tag{32}$$

so that *ex ante*

$$Pr\big(g\text{wins}\big) = \Phi\left(\sigma^{-1}\Lambda_{\mathcal{U}}\left[\sqrt{\frac{\Pi_g}{p_g}}\Phi^{-1}(\gamma_g) - \sqrt{\frac{\Pi_{-g}}{p_{-g}}}\Phi^{-1}(\gamma_{-g})\right]\right). \tag{33}$$

This calculation is general. With the assumption that $p_g = p_{-g} = p$ and $\gamma_g = \gamma_{-g} = \gamma$, I can write

$$Pr\big(g\text{wins}\big) = \Phi\left(\Lambda_{\mathcal{U}}\Phi^{-1}(\gamma)\frac{\sqrt{\Pi_g} - \sqrt{\Pi_{-g}}}{\sqrt{\Pi_g + \Pi_{-g} - 2p(1 - 1/\bar{P}) - 4\sqrt{p}}}\right). \tag{34}$$

The comparative statics results are easily derived from this expression. □

Proof of Proposition 6. I prove the result for the first case $\mathcal{K} \subset \mathcal{U}_+$. Suppose $\gamma < 1/2$ and fix an action P_g for the group leader of $-g$, then the best response of the group leader of g is the level of precision P_{-g} (remember that a group leader chooses the precision of the information revealed to the other group) that maximizes (34) given P_{-g}. The expression in (34) is strictly below $1/2$ for $P_{-g} < P_g$ because then $\Pi_{-g} < \Pi_g$ and Φ is evaluated at a negative point. The expression is strictly above $1/2$ for $P_{-g} > P_g$. The expression seen as a function of P_{-g} is continuous on a compact and is therefore maximized by some P_{-g} which must be strictly above P_g from the preceding argument. Because the former is true for any g with any initial P_g it implies that the only possible fixed point of the best-response dynamics is at $P_g = P_{-g} = \overline{K}$.

The other results are obtained by noticing that switching to either $\mathcal{K} \subset \mathcal{U}_-$ or $\gamma > 1/2$ changes the sign of the expression at which $\Phi(.)$ is evaluated in (34). □

Proof of Proposition 7. I can calculate the expected participation explicitly

$$l^e = \Phi\big(-\zeta^{-1}\Pi^{1/2}\Phi^{-1}(\gamma)\big), \tag{35}$$

where

$$\zeta^2 = p + \frac{1}{\left(P + \bar{P} - \sqrt{p}\right)^2}\left(\Pi^2\left(P + \bar{P}\right) + P\left(p^2 + p\right) + p(p+P)^2/\bar{P} + p^2\bar{P} - 2p^{5/2}\right).$$

I can rewrite $\zeta^{-1}\Pi^{1/2} = \sqrt{Q(P)/R(P)}$ as the square root of a fraction of two polynomials in P. In particular, this expression is always positive, and it is easy to verify that it goes to 1 when $P \to \infty$. The polynomial $Q(P)$ has a unique root in P, given by $P = \sqrt{p} - \bar{P}$, and $R(P)$ has no root in P since $\zeta^2 > 0$. Also, when $P = 0$, the expression is finite. Therefore it reaches a maximum when $P \to \infty$ and a minimum at $P = \sqrt{p} - \bar{P}$. It follows that when the cost is low ($\gamma < 1/2$), l^e is maximized at $P \to \infty$ and minimized at $P = \sqrt{p} - \bar{P}$ and the maximum expected total participation is $2(1-\gamma)$, that is everybody participates, while the opposite is true when the cost is high ($\gamma > 1/2$) and the maximum expected total participation is 1. When $\gamma = 1/2$, $l^e = \Phi(0) = 1/2$. □

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