Altruism in Networks

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Abstract: We provide the first analysis of altruism in networks. Agents are embedded in a fixed network and care about the well-being of their network neighbors. Depending on incomes, they may provide financial support to their poorer friends. We study the Nash equilibria of the resulting game of transfers. We show that equilibria maximize a concave potential function. We establish existence, uniqueness of equilibrium consumption and generic uniqueness of equilibrium transfers. We characterize the geometry of the network of transfers and highlight the key role played by transfer intermediaries. We then study comparative statics. A positive income shock to an individual benefits all. For small changes in incomes, agents in a component of the network of transfers act as if they were organized in an income-pooling community. A decrease in income inequality or expansion of the altruism network may increase consumption inequality.

Keywords: private transfers, altruism, social networks, neutrality, inequality.

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I Introduction

Private transfers play a significant role in our economies.[1] They act as major sources of redistribution and informal insurance, and interact in complex ways with public policies (Angelucci & De Giorgi 2009, Cox, Hansen & Jimenez 2004). They also seem to be motivated, to a large extent, by altruism.[2] Individuals give to others they care about and, in particular, to their family and friends in need.[3] Family and friendship ties generally form complex networks, and private transfers flow through networks of altruism.

In this paper, we provide the first analysis of altruism in networks. Agents are embedded in a fixed network and care about the well-being of their network neighbors. We adopt a benchmark model of altruism and assume that an agent's social utility is a linear combination of her private utility and others' private and social utilities. Depending on incomes, agents may provide financial support to their poorer friends. We study the Nash equilibria of this game of transfers.

We find that transfers and consumption depend on the network in complex ways. In equilibrium, an individual's transfers may be affected by distant agents. Income shocks may propagate throughout the network of altruism. Our analysis highlights the role played by transfer intermediaries, transmitting to poorer friends money received from richer friends, in mediating these effects.

We develop our analysis in two steps. We first uncover a key property of the game. We show that Nash equilibria maximize a concave potential function, linked to well-known problems of optimal transport on networks. We build on this reformulation of equilibrium conditions and establish existence, uniqueness of equilibrium consumption and generic uniqueness of equilibrium transfers. We then analyze the geometry of the network of transfers and its relation to the underlying network of altruism. We show that the transfer network contains no directed cycle and, generically, no undirected cycle. In other words, it

[1] This holds both in developing and in developed economies. For instance, remittances received in 2009 in the Philippines represent 12% of GDP (Worldbank 2011) while interhousehold transfers in the US in 2003 are estimated at 1.2% of GDP (Lee, Donehower & Miller 2011).

[2] See, e.g., Foster & Rosenzweig (2001), Leider et al. (2009), Ligon & Schechter (2012).

[3] See, e.g., Fafchamps & Gubert (2007), Fafchamps & Lund (2003), de Weerdt & Dercon (2006), de Weerdt & Fafchamps (2011).

is formed of disconnected trees. Furthermore, money must flow in equilibrium through the strongest paths of the altruism network. Intermediaries naturally appear when the altruism network is intransitive, for instance when agents do not care about their friends' friends.

Second, we study comparative statics with respect to incomes and to the altruism network. We show that equilibrium consumption varies monotonically with incomes. A positive income shock to an individual weakly benefits all other individuals. We then characterize the impact of small changes in the income profile. We find that this impact depends on the structure of equilibrium transfers before the change. Agents in a component of the initial network of transfers act as if they were organized in an income-pooling community. A small redistribution leaving components' aggregate incomes unchanged does not affect consumption. By contrast, an individual's consumption decreases when her component's aggregate income decreases. Redistributing resources away from rich benefactors of poor communities may then worsen outcomes for community members and increase inequality.

Finally, we characterize the impact of an increase in the strength of an altruistic link. This impact also depends on the structure of equilibrium transfers before the change. When an agent becomes more altruistic towards another agent, she tends to give him more and to consume less. This reduces the consumption of agents indirectly connected to her through transfer paths. By contrast, agents indirectly connected to the receiver gain. Depending on where this increase takes place, expansion of altruism can aggravate consumption inequality.

Our analysis introduces networks into the economics of altruism. Building on Barro (1974) and Becker (1975), economists have placed altruism at the heart of their study of family behavior. They have generally failed to recognize, however, that family ties form complex networks. Existing models are either static models with a few fully connected agents (e.g. Alger & Weibull (2010), Bernheim & Stark (1988), Bruce & Waldman (1991)) or dynamic models with disconnected dynastic families (e.g. Altig & Davis (1992), Hori & Kanaya (1989), Laitner (1988)). To our knowledge, the only exceptions are Bernheim & Bagwell (1988) and Laitner (1991). However, these two studies focus on the neutrality of public policies and do not characterize, as we do here, the nature and general properties of Nash equilibria.

We find that networks alter our understanding of altruistic behavior quite deeply.[4] We clarify the implications of different assumptions on altruistic preferences. With two agents, caring about the other's social utility is equivalent to caring about her private utility, see Bernheim & Stark (1998). We show that this equivalence breaks down under network interactions. When agents care about others' social utilities, they end up caring about their friends' friends. The resulting altruism network is transitive and intermediaries can always be bypassed in equilibrium. We also revisit the question of the neutrality of public policies under altruism. Extending earlier results of Barro (1974) and Becker (1974), Bernheim & Bagwell (1988) show that any small redistribution is neutral when the network of equilibrium transfers is connected.[5] We argue that this situation is unlikely to occur even with dense and strong altruistic ties. We show that neutrality fails to hold when the network of transfers is disconnected and characterize what happens in that case.

Our analysis also introduces altruism into the economics of networks, contributing to two strands of this fast-growing literature. The paper first advances the analysis of games played on fixed networks. We provide one of the first studies of a network game with multidimensional strategies,[6] unlike, for instance, Allouch (2015) who studies the private provision of a public good on a network.[7] A strategy profile is a vector of efforts in his context, but a network of transfers in ours. This increase in dimensionality is linked to deep differences in assumptions and outcomes. Actions are substitutes in Allouch (2015) and, to be neutral, a small redistribution must leave the income of every neighborhood unchanged. By contrast, the transfer game here involves a mixture of substitutes and complements and neutrality holds when the incomes of all components of the transfer network are unchanged.

[4]In reality, agents may be paternalistic (Pollak 1988), may derive a warm glow from giving (Andreoni 1989), or may care about how others reached their current situation (Alesina & Angeletos 2005). In future research, it would be interesting to study how these different kinds of altruism operate on networks. Our analysis can thus be viewed as the first step in a broader research program.

[5]This is related to neutrality results in models of private provision of multiple public goods, see Bergstrom, Blume & Varian (1986), Bernheim (1986), Cornes & Itaya (2010). A key difference, however, is that altruistic agents are not passive recipients and may transfer money themselves.

[6]Existing work mainly focuses on scalar strategies. Exceptions include Goyal, Konovalov & Moraga-González (2008) and Franke & Öztürk (2015).

[7]See Acemoglu, Garcia-Rimeno & Robinson (2015), Bramoullé & Kranton (2007a) and Bramoullé, Kranton & D'amours (2014) for related analyses.

Second, the paper contributes to the literature on private transfers in social networks. In particular, Ambrus, Mobius & Szeidl (2014) study risk-sharing when agents are embedded in a fixed, weighted network.[8] They assume that links serve as social collateral and characterize the Pareto-constrained risk-sharing arrangements. In our context, the network describes the structure of social preferences. Transfers are obtained as Nash equilibria of a non-cooperative game and generate redistribution even in the absence of risk.

II Setup

We consider a model of private transfers between $n \geq 2$ agents. Agent i has income $y_i^0 \geq 0$ and may give $t_{ij} \geq 0$ to agent j. The collection of bilateral transfers defines a network $\mathbf{T} \in \mathbb{R}_+^{n^2}$. By convention, $t_{ii} = 0$. Income after transfers, or consumption, y_i is equal to

$$y_i = y_i^0 - \sum_j t_{ij} + \sum_k t_{ki} \tag{1}$$

Thus, private transfers redistribute income across agents and aggregate income is conserved $\sum_i y_i = \sum_i y_i^0$.

We assume that agents care about each other. Preferences have a private and a social component. Agent i's private preferences are represented by utility function $u_i : \mathbb{R} \rightarrow \mathbb{R}$. We assume that u_i is twice differentiable and satisfies $u_i' > 0$, $u_i'' < 0$ and $\lim_{y \rightarrow \infty} u_i'(y) = 0$. Following the economic literature on the family, we assume that agents may *a priori* care about others' private and social utilities. Agent i's social preferences are represented by utility function $v_i : \mathbb{R}^n \rightarrow \mathbb{R}$ such that:

$$v_i(\mathbf{y}) = u_i(y_i) + \sum_{j \neq i} a_{ij} u_j(y_j) + \sum_{j \neq i} b_{ij} v_j(\mathbf{y}) \tag{2}$$

where $a_{ij}, b_{ij} \geq 0$ represent primitive preference parameters.

Social utilities in (2) are implicitly defined as solutions of a system of equations. As in

[8] Bloch, Genicot & Ray (2008) and Bramoullé & Kranton (2007b) study network stability in risk-sharing contexts.

Bergstrom (1999), this system has a unique well-behaved solution if and only if $\lambda_{\max}(\boldsymbol{B}) < 1$ where $\lambda_{\max}(\boldsymbol{B})$ denotes $\boldsymbol{B}$'s largest eigenvalue. The matrix $\mathbf{M} = (\mathbf{I} - \mathbf{B})^{-1}(\mathbf{I} + \mathbf{A})$ then has nonnegative elements and social utilities are equal to $\mathbf{v} = \mathbf{Mu}$. Letting $\alpha_{ij} = m_{ij}/m_{ii}$, we can represent agents' social preferences in the following reduced-form:[9]

$$v_i(\mathbf{y}) = u_i(y_i) + \sum_{j \neq i} \alpha_{ij} u_j(y_j) \tag{3}$$

We assume further that $\alpha_{ij} < 1$, so that an agent values her own private utility more than any other agent's private utility. The collection of bilateral coefficients α_{ij} defines the *altruism network* $\boldsymbol{\alpha}$. By convention $\alpha_{ii} = 0$. When $\alpha_{ij} > 0$, i ultimately cares about j's private well-being and the size of the coefficient measures the strength of the altruistic tie.

Caring about others' social utilities, as in (2), implies caring about others' private utilities, as in (3). In general, many different primitive preferences can lead to the same reduced-form preferences. In particular, we say that *an altruism network is consistent with deferential caring* if there exist underlying primitive preferences where agents only care about others' social utilities (Pollak 2003). Formally, this holds when there exists $\mathbf{B} \geq \mathbf{0}$ such that $b_{ii} = 0$, $\lambda_{\max}(\mathbf{B}) < 1$ and $\alpha_{ij} = m_{ij}/m_{ii}$ with $\mathbf{M} = (\mathbf{I} - \mathbf{B})^{-1}$. We will see that deferential caring induces specific restrictions on the shape of the altruism network and on giving behavior.

We make the following joint assumption on private utilities and altruistic coefficients

$$\forall i, j, \forall y, u_i'(y) > \alpha_{ij} u_j'(y) \tag{4}$$

This condition guarantees that an agent's gift to a friend never makes this friend richer than her. Indeed, when agent i plays a best-response, she chooses her transfers to j to equalize her marginal utility $u_i'(y_i)$ and the discounted marginal utility of j, $\alpha_{ij} u_j'(y_j)$. Therefore, $t_{ij} > 0 \Rightarrow y_i > y_j$. In particular, an agent never gives away all her income and the budget constraint $y_i \geq 0$ is always satisfied in equilibrium.

The collection of agents, transfers and altruistic utilities defines a simultaneous game.

[9] Since $m_{ii} > 0$, v_i and v_i/m_{ii} represent the same preferences.

Our main objective is to study the Nash equilibria of this game and how equilibrium transfers $\mathbf{T}$ and consumption $\mathbf{y}$ depend on incomes $\mathbf{y}^0$ and on the altruism network $\boldsymbol{\alpha}$. In equilibrium, each agent chooses her transfers to maximize her altruistic utility conditional on transfers made by others.

The transfer game exhibits a complex pattern of strategic interactions and externalities. An agent tends to reduce her transfer to a friend when this friend receives more transfers from others and to increase her transfer when her friend makes more transfers herself. Thus, transfers to an agent from different givers are strategic substitutes while transfers to and from an agent are strategic complements. An agent also suffers a loss in utility from her friend's transfers to others, but benefits from the transfers her friend receives. Externalities may be positive or negative, and the externality pattern is rooted in the structure of the altruism network. These externalities imply that Nash equilibria are typically not Pareto-optima. A well-known exception is a situation where one agent makes transfers to all the others (Becker 1974, Arrow 1981). We further discuss the misalignment between equilibrium behavior and welfare in Section III.

III Equilibrium analysis

In this section, we describe key properties of Nash equilibria. We show that equilibria are the solutions to the problem of maximizing a concave potential function. Building on this reformulation, we establish existence, uniqueness of consumption and generic uniqueness of transfers and we characterize the geometric structure of the network of transfers.

Let us first introduce a few notions and notations. Let $\mathbf{T}_i$ and $\mathbf{T}_{-i}$ denote transfers made by i and by agents other than i. Let $S = \left\{\mathbf{T} \in \mathbb{R}_+^{n^2} \colon \alpha_{ij} = 0 \Rightarrow t_{ij} = 0\right\}$ be the set of networks of transfers where agents only give to others they care about. If $\alpha_{ij} > 0$, define $c_{ij} = -\ln(\alpha_{ij})$ which we refer to as a transfer cost over the link (i, j). This is a virtual rather than an actual cost which is lower when the altruistic link is stronger. The graph of transfers is the binary graph where i is connected to j if $t_{ij} > 0$. A path connecting i and j in $\mathbf{T}$ is a set of distinct agents $i_1 = i$, i_2,..., $i_{l+1} = j$ such that $t_{i_1 i_2} > 0$,..., $t_{i_l i_{l+1}} > 0$. A cycle is a set of agents $i_1 = i$, i_2,..., $i_{l+1} = i$ such that i_1,..., i_l form a path and $t_{i_l i_{l+1}} > 0$.

An undirected path is a path of the undirected graph where i is linked with j when $t_{ij} > 0$ or $t_{ji} > 0$, and similarly for an undirected cycle. Network $\mathbf{T}$ is acyclic when it has no cycle and is a forest when it has no undirected cycle. The cost of path i_1, i_2,..., i_{l+1} in $\boldsymbol{\alpha}$ is equal to $\sum_{s=1}^{l} c_{i_s i_{s+1}}$. A least-cost path connecting i to j in $\boldsymbol{\alpha}$ has the lowest cost among all paths connecting both agents.

Since v_i is concave as a function of $\mathbf{T}_i$ for any $\mathbf{T}_{-i}$, the first-order conditions of i's utility maximization are necessary and sufficient. Therefore, a network of transfers $\mathbf{T}$ is a Nash equilibrium if and only if the following conditions are satisfied:

$$\forall i, j, u_i'(y_i) \geq \alpha_{ij} u_j'(y_j) \text{ and } t_{ij} > 0 \Rightarrow u_i'(y_i) = \alpha_{ij} u_j'(y_j) \tag{5}$$

In particular, $\alpha_{ij} = 0 \Rightarrow t_{ij} = 0$. Agents only give to others they care about. Together with (4), these conditions imply that consumption levels decrease along any path of the transfer network. In particular, transfer networks must be acyclic in equilibrium.

To illustrate, suppose that agents have homogenous CARA utilities $u_i(y) = -e^{-Ay}/A$. Conditions (5) become: $\forall i, j, y_i \leq y_j + c_{ij}/A$ and $t_{ij} > 0 \Rightarrow y_i = y_j + c_{ij}/A$. The difference in consumption between a richer agent i and a poorer friend j has an upper bound which is proportional to c_{ij}, and this bound is attained whenever a transfer is made.

Interestingly, we can view equilibria of the transfer game as solutions to a social planner's problem with concave objective function

$$\varphi(\mathbf{T}) = \sum_i \int_1^{y_i} \ln(u_i'(x))dx - \sum_{i,j:\alpha_{ij}>0} c_{ij} t_{ij} \tag{6}$$

Indeed, note that $\partial\varphi/\partial t_{ij} = -\ln(u_i'(y_i)) + \ln(u_j'(y_j)) + \ln(\alpha_{ij})$. Thus, the first-order conditions of the problem of maximizing φ over S are equivalent to equilibrium conditions (5). In fact, φ is a best-response potential for the transfer game since i's best-response to $\mathbf{T}_{-i}$ is exactly $\arg\max_{\mathbf{T}_i} \varphi(\mathbf{T})$ (Voorneveld 2000).

Therefore, agents act as if they are all trying to maximize φ. The potential can be viewed as the difference between benefits $B(\mathbf{y}) = \sum_i \int_1^{y_i} \ln(u_i'(x))dx$ and virtual costs $\sum_{i,j:\alpha_{ij}>0} c_{ij} t_{ij}$. The function B is related to the utilitarian social welfare $W(\mathbf{y}) = \sum_i u_i(y_i)$.

Let $\mathbf{y}^*$ be the utilitarian optimum which maximizes W over all redistributions. This allocation equalizes marginal utilities across all agents, and hence maximizes B as well. Thus, B and W are two strictly concave functions with the same maximum and B tends to be higher when equilibrium consumption is closer to the utilitarian optimum. For instance under common CARA utilities, $B(\mathbf{y}) = B(\mathbf{y}^*) - \frac{1}{2}AnVar(\mathbf{y})$ where $Var(\mathbf{y})$ denotes the variance of the consumption profile. The utilitarian optimum then corresponds to equal income-sharing and B is higher when consumption variance is lower.

Nash equilibria generally do not maximize welfare, however, because of the second term in the potential. If we interpret c_{ij} as the cost of transferring 1 unit of money from i to j, then this term $\sum_{i,j:\alpha_{ij}>0} c_{ij}t_{ij}$ represents the overall cost of transfers $\mathbf{T}$. In particular, the potential property implies that *equilibrium transfers minimize the overall cost of reaching* $\mathbf{y}$ *from* $\mathbf{y}^0$. This turns out to be a classical problem of optimal transportation on networks, known as "minimum-cost flow", with well-known implications (Ahuja, Magnanti & Orlin 1993, Galichon 2016). In particular, it implies that transfers flow through least-cost paths of the altruism network. Indeed, if some money flows from i to j through a path that does not have the lowest cost, we can reduce transfer costs without altering consumption by redirecting transfers through a least-cost path. It also provides another explanation for the acyclicity of transfer networks, as eliminating a cycle reduces transfer costs without changing consumption.

Together with assumption (4), the acyclicity of transfer networks implies that the consumption distribution second-order stochastically dominates the income distribution. Indeed, consumption can be obtained from incomes via bilateral Pigou-Dalton redistributions from richer to poorer agents. Consumption inequality is thus lower than income inequality.

We assemble these properties and further implications of the potential in the following theorem. A property is said to hold generically if the set on which it does not hold has measure zero. Proofs are provided in the Appendix, except when stated otherwise.

Theorem 1 *A network of transfers* $\mathbf{T}$ *is a Nash equilibrium if and only if* $\mathbf{T}$ *maximizes the concave function* φ *over* S*. A Nash equilibrium exists. Equilibrium transfers are acyclic and flow through least-cost paths of* $\boldsymbol{\alpha}$*. The profile of equilibrium consumption* $\mathbf{y}$ *is unique,* *continuous in* $\mathbf{y}^0$ *and* $\boldsymbol{\alpha}$*, and second-order stochastically dominates* $\mathbf{y}^0$*. Generically in* $\boldsymbol{\alpha}$*, the network of equilibrium transfers is unique and is a forest.*

We briefly comment on the more technical parts of the theorem. We show that we can restrict attention to bounded transfers, leading to existence.[10] To prove uniqueness of consumption, we express the potential as a function of consumption only and show that this reformulated potential is strictly concave in $\mathbf{y}$. This extends the result obtained by Arrow (1981) for groups to networks.[11] Continuity follows from an application of the maximum theorem. Finally, we prove the generic results through a thorough analysis of the problem of cost-minimization in Supplementary Appendix A. We show that under multiplicity, some equilibrium transfer network must have an undirected cycle and that this can only happen non-generically.

Theorem 1 shows that equilibrium determination falls within the domain of convex optimization. We can thus adapt classical algorithms to compute Nash equilibria in practice (Bertsekas 2015). In particular, the potential cannot decrease when one agent plays a best response. We show in Supplementary Appendix B that under uniqueness, sequences of asynchronous best-responses converge to the equilibrium. We make use of this property in our numerical simulations below.

Can we further characterize Nash equilibria and their architecture? The least-cost property reveals a tight relationship between the network of altruism and the network of transfers. We next explore some of its implications. Note first that some altruistic links are never activated. To formalize this property, introduce the *transitive closure* of the altruism network, $\hat{\boldsymbol{\alpha}}$, as follows: $\hat{\alpha}_{ij} = \Pi_{s=1}^{l} \alpha_{i_s i_{s+1}}$ if i_1, i_2,..., i_{l+1} is a least-cost path connecting i to j and $\hat{\alpha}_{ij} = 0$ if i is not connected to j through a path in $\boldsymbol{\alpha}$. Agents who are indirectly connected in $\boldsymbol{\alpha}$ are directly connected in $\hat{\boldsymbol{\alpha}}$. A network is transitive if $\boldsymbol{\alpha} = \hat{\boldsymbol{\alpha}}$. Theorem 1 then implies that $t_{ij} = 0$ in any equilibrium if $\alpha_{ij} < \hat{\alpha}_{ij}$. When the direct link between i and j is weaker than an indirect connection, money never flows directly from i to j.[12]

[10] Alternatively, existence follows from Corollary 2 in Mercier Ythier (2006).

[11] Arrow (1981) assumes that $v_i(\mathbf{y}) = u_i(y_i) + \sum_{j \neq i} w(y_j)$. This corresponds to formulation (3) when the altruism network is complete, i.e., $\forall i \neq j, \alpha_{ij} = \alpha$, $u_i = u$ and $w = \alpha u$.

[12] Conversely, there exists an equilibrium with $t_{ij} > 0$ if $\alpha_{ij} = \hat{\alpha}_{ij} > 0$.

In some cases, the graph of transfers can be fully determined by the least-cost property. Consider, for instance, a connected altruism network with a rich benefactor. Suppose that agent i has much higher income than anyone else. Money then flows from this rich benefactor to all other agents. The generic condition identified in Theorem 1 guarantees that there is a unique least-cost path connecting i to any j. All links in these least-cost paths are activated and allow financial support to trickle down from the rich benefactor to distant agents.[13] The following example illustrates.

Example 1 *Five agents are connected through an altruistic network depicted in the Left panel of Figure 1, with links of different intensities. The Right panel depicts the graph of transfers in equilibrium when agent* 1 *has high income. The direct link between* 1 *and* 3 *is weaker than their indirect connection through* 2*, hence money does not flow directly from* 1 *to* 3*. There are two paths connecting* 2 *to* 5*, and transfers flow through the stronger, or least-cost, path* $2-4-5$.

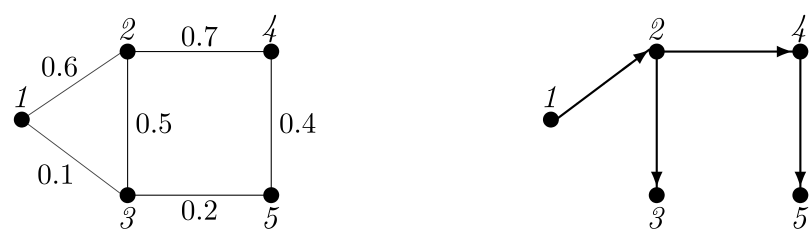

Figure 1: The graph of transfers with a rich benefactor.

This example illustrates the key role played by *transfer intermediaries*, i.e., agents who both give and receive in equilibrium. These agents allow money to flow from richer to poorer parts of society. From Theorem 1, we see that transfer intermediaries can only appear when friends of friends have sufficiently weak direct ties. When $t_{ij} > 0$ and $t_{jk} > 0$ in equilibrium, then $\alpha_{ik} < \hat{\alpha}_{ik} = \alpha_{ij}\alpha_{jk}$ and the direct link between i and k is weaker than their indirect connection through j. We show next that this condition is, in fact, necessary and sufficient.

[13]Formally, the graph of transfers is then a directed spanning tree minimizing the sum, over j, of the costs of the paths connecting i to j.

Theorem 2 *There exists a Nash equilibrium without transfer intermediary for every* $\mathbf{y}^0$ *if and only if the altruism network* $\boldsymbol{\alpha}$ *is transitive. This holds whenever* $\boldsymbol{\alpha}$ *is consistent with deferential caring.*

To prove Theorem 2, we develop a constructive procedure which, starting from any Nash equilibrium, builds an equilibrium without transfer intermediaries, see Supplementary Appendix C. The idea is to redirect through direct links the transfers originally flowing through indirect links. This can be done while respecting equilibrium conditions precisely when the network is transitive.

Theorem 2 also shows that transfer intermediaries generally do not emerge under deferential caring.[14] To see why, suppose that i cares about v_j and j cares about v_k. Agent i then internalizes the fact that her friend j is herself altruistic. In the reduced-form preferences, v_i ultimately depends on u_k. The altruism network induced by deferential caring is thus transitive. With two agents, caring about the other's private or social utility yield equivalent formulations, a fact long noted by researchers (Bernheim & Stark 1988). Theorem 2 shows that this equivalence breaks down under network interactions. Caring about others' social utilities only leads to strong restrictions on the structure of reduced-form preferences.

Theorems 1 and 2 have empirical implications and may help inform the debate on the motives behind private transfers. Applied researchers have started to collect detailed information on transfers (Fafchamps & Lund (2003), de Weerdt & Fafchamps (2011)). Acyclicity and the forest structure provide testable implications that are easy to check given data on $\mathbf{T}$. Within our framework, the least-cost property allows researchers to infer information on the altruism network from observed transfers, even without information on private utilities. Under assumptions that guarantee equilibrium uniqueness, the presence of transfer intermediaries in the data implies that the altruism network is not transitive and that social preferences are not consistent with deferential caring.[15]

[14] By contrast, an altruistic network may be transitive without being consistent with deferential caring, see Supplementary Appendix C.

[15] Bringing the model to data would of course raise a number of issues including stochastic and dynamic aspects and transaction costs.

IV Comparative statics

In this section, we study how changes in incomes and in the altruism network affect consumption. We analyze how an altruistic society responds to individual income shocks and how public policies redistributing income across agents may be altered by private transfers. We also analyze how changes in altruistic preferences affect consumption. These effects are complex, and we show that a reduction in income inequality or an increase in altruism may end up creating more inequality.

A Changes in incomes

Our comparative statics result on incomes has two parts. First, we show that consumption is weakly increasing with incomes. A positive income shock to an agent weakly benefits everyone. Second, we characterize how consumption varies locally with incomes. This allows us to consider more complex changes, such as small redistributions. Understanding these effects requires a description of the generically unique equilibrium $\mathbf{T}$ generated by initial incomes $\mathbf{y}^0$. Denote by C_i the *component* of agent i in $\mathbf{T}$. This set contains i and agents connected to i through an undirected path in $\mathbf{T}$. For any profile of incomes $\tilde{\mathbf{y}}^0$ and subset C, let $\tilde{y}^0(C) = \sum_{j \in C} \tilde{y}^0_j$ denote the aggregate income of agents in C.

Theorem 3 *Equilibrium consumption y_i is increasing in y^0_i and weakly increasing in y^0_j for any $j \neq i$. Generically in $\boldsymbol{\alpha}$ and $\mathbf{y}^0$, there exists a neighborhood $\mathcal{V}$ of $\mathbf{y}^0$ and an increasing continuous function f_i for every i such that $\forall \tilde{\mathbf{y}}^0 \in \mathcal{V}$, $\tilde{y}_i = f_i(\tilde{y}^0(C_i))$.*

The monotonicity result seems intuitive: positive or negative shocks on individuals are absorbed by the whole network. To see this, consider incomes $\tilde{\mathbf{y}}^0$ that differ from $\mathbf{y}^0$ only in that $\tilde{y}^0_i > y^0_i$. Then let $U = \{j : \tilde{y}_j < y_j\}$ be the set of agents that are negatively affected by this positive shock on i's income. Suppose by contradiction that U is nonempty, and let $j \in U$. Then, for any agent k such that $\tilde{t}_{jk} > 0$, equilibrium conditions (5) imply that $\alpha_{jk} u'_k(y_k) \leq u'_j(y_j) < u'_j(\tilde{y}_j) = \alpha_{jk} u'_k(\tilde{y}_k)$, where the strict inequality follows from the definition of U. Hence, it must be the case that $k \in U$. Similarly, for any agent l such that $t_{lj} > 0$, equilibrium conditions imply that $u'_l(y_l) = \alpha_{lj} u'_j(y_j) < \alpha_{lj} u'_j(\tilde{y}_j) \leq u'_l(\tilde{y}_l)$, and therefore $l \in U$. To summarize, no money flows out of U in $\tilde{\mathbf{T}}$ and into U in $\mathbf{T}$. This implies that $\tilde{y}^0(U) \leq \tilde{y}(U) < y(U) \leq y^0(U)$, where the strict inequality comes from the definition U. This is impossible, however, since all incomes are weakly higher in $\tilde{\mathbf{y}}^0$.

The second result characterizes the effect of small changes in incomes on consumption. This effect depends on the structure of equilibrium transfers before the change. Everything works as if the components of this transfer network constituted income-pooling communities. In particular, *a small redistribution is neutral if and only if it does not redistribute income across components of the initial network of transfers.* Small redistributions within components leave consumption unaffected. Changes in incomes are then offset by adjustments in private transfers. More generally, the consumption of an agent increases if and only if the aggregate income of her component has increased, and even though her own income may have decreased.

To understand the result, note first that we focus on altruism networks that generate a unique equilibrium, hence the genericity in $\boldsymbol{\alpha}$. We then show that generically in $\mathbf{y}^0$, the graph of transfers is locally invariant in incomes. Indeed, by continuity, $t_{ij} > 0 \Rightarrow \tilde{t}_{ij} > 0$ and $u_i'(y_i) > \alpha_{ij} u_j'(y_j) \Rightarrow u_i'(\tilde{y}_i) > \alpha_{ij} u_j'(\tilde{y}_j)$ if $\tilde{\mathbf{y}}^0$ is close to $\mathbf{y}^0$. Hence the graph of transfers may be affected by small changes in incomes only when $t_{ij} = 0$ and $u_i'(y_i) = \alpha_{ij} u_j'(y_j)$. In that case, the link between i and j is on the edge of activation. It may or may not be activated depending on the direction of the income change. We show in Supplementary Appendix D that such situations are non-generic in $\mathbf{y}^0$. Finally, pick a connected component C and some $i \in C$. Because of the forest structure of transfer networks, equilibrium conditions (5) imply that, for any income $\tilde{\mathbf{y}}^0$ in a neighborhood of $\mathbf{y}^0$, the marginal utility of any agent $j \in C$ is proportional to the marginal utility of i, with a coefficient that only depends on $\boldsymbol{\alpha}$. The consumption of any agent in C, and hence aggregate consumption in C, can then be written as an increasing function of i's consumption. Since aggregate consumption is equal to aggregate income within components, i's consumption can be written as an increasing function of C's income. The following example illustrates.

Example 2 *Suppose that $C = \{i, j, k\}$, i gives to j and j gives to k. Consider common CARA utilities with $A = 1$. Consumption levels solve three equations, two obtained from* *conditions (5):* $y_i = y_j + c_{ij}$*,* $y_j = y_k + c_{jk}$ *and the conservation of income within* C*:* $y_i + y_j + y_k = y^0(C)$*. Solving these equations yields increasing functions of the component's income:* $y_i = \frac{1}{3}y^0(C) + \frac{2}{3}c_{ij} + \frac{1}{3}c_{jk}$*,* $y_j = \frac{1}{3}y^0(C) - \frac{1}{3}c_{ij} + \frac{1}{3}c_{jk}$ *and* $y_k = \frac{1}{3}y^0(C) - \frac{1}{3}c_{ij} - \frac{2}{3}c_{jk}$*.*

Theorem 3 extends, in our context, the result of Bernheim & Bagwell (1988) showing that any small redistribution is neutral when the network of transfers is connected. Such situations seem to be rare, however, in practice. We investigated this issue through extensive numerical simulations. For instance, consider 20 agents with common utilities $u_i(y) = \ln(y)$. Pick α_{ij} uniformly at random between 0.25 and 0.75 and y_i^0 uniformly at random between 0 and 1000. Over 1000 runs, the network of transfers is connected in only 0.6% of the runs.[16] Therefore even with dense altruism networks of strong ties, the network of transfers is generally not connected. Small redistributions between components are therefore not neutral.

In particular, the poorest agent's consumption drops if her component's income is reduced. A reduction in income inequality may thus increase consumption inequality, as shown in the following example.

Figure 2: Reducing income inequality may increase consumption inequality.

Example 3 *Three agents, depicted in Figure 2, have common CARA utilities with* $c_{ij}/A = 2$*. The Left Panel depicts initial incomes* $\mathbf{y}^0$ *and consumption* $\mathbf{y}$*; the Right Panel depicts redistribution* $\tilde{\mathbf{y}}^0$ *and consumption* $\tilde{\mathbf{y}}$*. The redistribution decreases income inequality by*

[16] The network of transfers has, on average, 2.7 isolated agents and at least two components with more than 2 agents in 93.5% of the runs. We ran simulations under a variety of assumptions. For instance, when incomes follow a Pareto distribution with minimum value 100 and tail index 1.16, the network of transfers is connected in only 16.7% of the runs.

transferring money from the richest to poorer agents. However, it ends up reducing consumption of the poorest and aggravating consumption inequality in the sense of second-order stochastic dominance.

We show in Supplementary Appendix D that the logic of the example generalizes. Consider altruism networks composed of two separate communities. When the difference in communities' incomes is high enough, any redistribution from a rich agent in the poor community to a poor agent in the rich community increases consumption inequality.

B Changes in the altruism network

We finally study how consumption varies with the altruism network. A change in one part of the network may have far-reaching repercussions. We identify who gains and who loses from a change in the intensity of an altruistic tie. Intuitively, if i becomes more altruistic towards j, i will consume less and j will consume more. But all agents that j was already making transfers to will also benefit, as j should give them more, and all those who were making transfers to j will be able to give less to j. This logic should extend to all agents who are initially connected to j by transfer paths that do not go through i. Our result shows that this intuition indeed holds. To see that, we consider an increase in α_{ij} holding other links unchanged. We say that the change is effective if it affects consumption $\tilde{\mathbf{y}} \neq \mathbf{y}$. Generically in $\boldsymbol{\alpha}$, the initial equilibrium $\mathbf{T}$ is a forest and we define the subcomponent $S_i(\mathbf{T})$ of i in $\mathbf{T}$ as the component of i in the network obtained from $\mathbf{T}$ by setting $t_{ij} = 0$, and similarly for j. Given two networks of transfers $\mathbf{T}$ and $\tilde{\mathbf{T}}$, denote by $\mathbf{T} \cap \tilde{\mathbf{T}}$ the graph such that $g_{ij} = 1$ if $t_{ij} > 0$ and $\tilde{t}_{ij} > 0$.

Theorem 4 *Generically in $\boldsymbol{\alpha}$ and $\mathbf{y}^0$, a small effective increase in α_{ij} is such that $S_i(\mathbf{T}) = \{k : \tilde{y}_k < y_k\}$ and $S_j(\mathbf{T}) = \{k : \tilde{y}_k > y_k\}$. An effective increase in α_{ij} is such that $S_i(\mathbf{T} \cap \tilde{\mathbf{T}}) \subset \{k : \tilde{y}_k < y_k\}$ and $S_j(\mathbf{T} \cap \tilde{\mathbf{T}}) \subset \{k : \tilde{y}_k > y_k\}$.*

Thus a small increase in altruism decreases the consumption of the giver and increases the consumption of the receiver,[17] but also decreases the consumption of every agent indi-

[17] The increase is only effective when $\tilde{t}_{ij} > 0$.

rectly connected to the giver and increases the consumption of every agent indirectly connected to the receiver. This characterization partially extends to large increases, through the graph of transfers which are positive both before and after the change. Depending on the shape of the network of transfers, this may reduce the consumption of the poorest and increase inequality.[18] The following example illustrates.

Example 4 *Six agents, depicted in Figure 3, have common CARA utilities with* $c_{ij}/A = 3$*. The Left Panel depicts the original network* $\boldsymbol{\alpha}$*, formed of two separate lines. The Middle Panel depicts equilibrium* $\mathbf{T}$ *and consumption* $\mathbf{y}$*. The Right Panel depicts equilbrium* $\tilde{\mathbf{T}}$ *and consumption* $\tilde{\mathbf{y}}$ *in network* $\tilde{\boldsymbol{\alpha}}$ *where a new connection is added between the richest agent on the left and the poorest agent on the right. The new connection increases consumption of agents on the right, to the detriment of agents on the left, and increases inequality in the sense of second-order stochastic dominance.*[19]

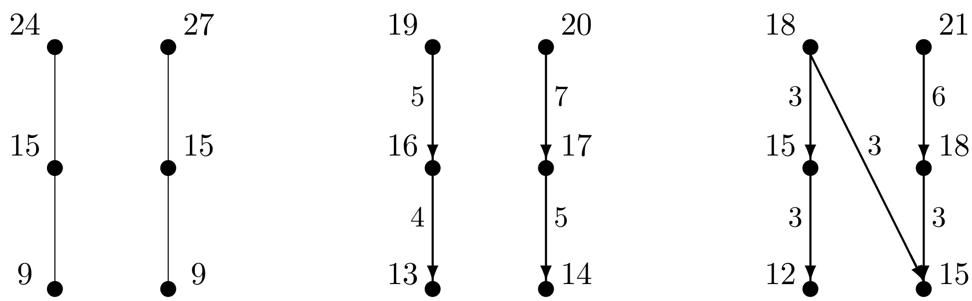

Figure 3: An expansion of the altruism network can increase inequality.

Thus, an expansion in the altruism network may increase, or decrease, inequality depending on where this expansion takes place. As with changes in incomes, these impacts critically depend on the the structure of the network of transfers before the change. Undirected transfer paths create linkages between agents. Theorems 3 and 4 explore implications of these linkages for comparative statics.

[18] Consumption y_k may also vary non-monotonically in α_{ij}, see Supplementary Appendix D.

[19] The new distribution $(12, 15, 15, 18, 18, 21)$ can be obtained from the original distribution $(13, 14, 16, 17, 19, 20)$ by transferring 1 from 13 to 14, 16 to 17 and 19 to 20.

APPENDIX

Proof of Theorem 1. Arguments in the text prove the potential and least-cost path properties. Acyclicity and the budget constraints, $\sum_{j\neq i} t_{ij} - t_{ji} \leq y_i^0$ for each i, together imply that no transfer can exceed aggregate income $\sum_k y_k^0$. We can then rewrite the set of Nash equilibria as $\arg\max_{\mathbf{T}\in S'} \varphi(\mathbf{T})$, where $S' = \{\mathbf{T} \in S : \forall i, j, t_{ij} \leq \sum_k y_k^0\}$. This set is closed and bounded, and hence compact. Since φ is continuous, a Nash equilibrium exists.

For the uniqueness of equilibrium consumption, let

$$c(\mathbf{y}, \mathbf{y}^0) = \min_{\mathbf{T}\in S} \sum_{(i,j):\alpha_{ij}>0} t_{ij}c_{ij} \quad \text{s.t.} \ \ \forall i, \ y_i^0 + \sum_{j\neq i}(t_{ji} - t_{ij}) = y_i,$$

be the value function of the cost minimization program associated with the potential. $c(\mathbf{y}, \mathbf{y}^0)$ is continuous. By duality, it is also convex in $\mathbf{y}$ as the value function of the dual problem, which is a linear minimization program in $\mathbf{y}$. Now note that we can rewrite the problem of finding consumption as $\max_{\mathbf{y}} \ B(\mathbf{y}) - c(\mathbf{y}, \mathbf{y}^0)$. $B(\mathbf{y})$ being strictly concave and $c(\cdot, \mathbf{y}^0)$ convex, this program has a unique solution.

For the second-order stochastic dominance property, we show that consumption can be obtained from incomes through a series of Pigou-Dalton transfers from richer to poorer agents. Consider an equilibrium $\mathbf{T}$. By acyclicity, there is an agent i who does not receive. From the initial incomes, apply i's transfers first, in any order. Then remove i and repeat until there is no transfers left. This procedure leads to an ordering of all pairwise transfers and hence yields equilibrium consumption. This ordering also guarantees that a transfer always takes place from a richer to a poorer agent.

The proof of the generic uniqueness and forest structure of equilibrium transfers is derived in Supplementary Appendix A. □

The proof of local results in Theorems 3 and 4 relies on the following lemma

Lemma 1 *Generically in $(\boldsymbol{\alpha}, \mathbf{y}^0)$, there exists a neighborhood $\mathcal{V}$ of $(\boldsymbol{\alpha}, \mathbf{y}^0)$, such that for every $(\tilde{\boldsymbol{\alpha}}, \tilde{\mathbf{y}}^0) \in \mathcal{V}$, the (unique) equilibrium transfer networks $\mathbf{T}$ and $\tilde{\mathbf{T}}$ have the same graph, i.e. for every (i,j), $t_{ij} > 0 \Leftrightarrow \tilde{t}_{ij} > 0$.*

Proof of Lemma 1. First, by Theorem 1, we consider only generic $\boldsymbol{\alpha}$ that lead to a unique equilibrium for every $\mathbf{y}^0$. Second, we choose $(\boldsymbol{\alpha}, \mathbf{y}^0)$ so that in the corresponding equilibrium, $t_{ij} = 0 \Leftrightarrow u_i'(y_i) > \alpha_{ij}u_j'(y_j)$. We show in Supplementary Appendix D that this property is generic, and, therefore, holds in a neighborhood of $(\boldsymbol{\alpha}, \mathbf{y}^0)$. By the maximum theorem, both equilibrium transfers and consumption is locally continuous at $(\boldsymbol{\alpha}, \mathbf{y}^0)$. Therefore, there exists a neighborhood $\mathcal{V}$ of $(\boldsymbol{\alpha}, \mathbf{y}^0)$, such that for every $(\tilde{\boldsymbol{\alpha}}, \tilde{\mathbf{y}}^0) \in \mathcal{V}$, we have $t_{ij} > 0 \Rightarrow \tilde{t}_{ij}$, and $t_{ij} = 0 \Leftrightarrow u_i'(y_i) < \alpha_{ij}u_j'(y_j) \Rightarrow u_i'(\tilde{y}_i) < \alpha_{ij}u_j'(\tilde{y}_j) \Leftrightarrow \tilde{t}_{ij} = 0$. □

Proof of Theorem 3. The fact that all agents benefit from a positive income shock to i is proved in detail below the theorem. To show that i benefits strictly, suppose that $\tilde{\mathbf{y}}^0$ and $\mathbf{y}^0$ differ only in that $\tilde{y}_i^0 > y_i$, and let $V = \{j : \tilde{y}_j > y_j\} \neq \varnothing$. By the same argument as above, $y^0(V) \leq y(V) < \tilde{y}(V) \leq \tilde{y}^0(V)$. But since i is the only agent whose income strictly increases, it must be that $i \in V$.

Then by Lemma 1, we can consider a generic $(\boldsymbol{\alpha}, \mathbf{y}^0)$ such that the graph of the unique equilibrium transfer network is constant over a neighborhood $\mathcal{V}$ of $\mathbf{y}^0$. Let i and j be in the same connected component C of $\mathbf{T}$. Then there exists a unique path $i = i_0, i_1, \cdots, i_\ell = j$ of distinct agents such that, for every $k = 0, \cdots, \ell - 1$, $t_{i_k i_{k+1}} > 0$ or $t_{i_{k+1} i_k} > 0$. For each k, let $\beta_{i_k i_{k+1}} = \alpha_{i_k i_{k+1}} > 0$ in the first case, and $\beta_{i_k i_{k+1}} = \alpha^{-1}_{i_{k+1} i_k} > 0$ in the second case. Equilibrium conditions (5) imply that we can write $u'_j(y_j) = u'_i(y_i) / \prod_{k=0}^{\ell-1} \beta_{i_k i_{k+1}}$. Since i and j were chosen arbitrarily in C, this implies that we can write the consumption of any agent j in C as an increasing function $g_j(y_i)$ of i's final consumption. The function g_j only depends on the altruism network $\boldsymbol{\alpha}$, and, by construction, this relationship also holds for any alternative income profile $\tilde{\mathbf{y}}^0 \in \mathcal{V}$. Then, for any such $\tilde{\mathbf{y}}^0$, we can write that aggregate income in C, $\tilde{y}^0(C)$, is equal to aggregate consumption in C because, C being a connected component of the transfer network, no money flows in or out of C. Hence $\sum_{j \in C} g_j(\tilde{y}_i) = \tilde{y}^0(C)$. Since the left-hand side is an increasing function of $\tilde{y}_i$, it shows that $\tilde{y}_i$ can be written as an increasing function of $\tilde{y}^0(C)$. $\square$

Proof of Theorem 4. By the same continuity argument behind Lemma 1, we can pick a generic $(\boldsymbol{\alpha}, \mathbf{y}^0)$ and a neighborhood $\mathcal{V}$ of α_{ij} such that the graphs of transfers for $(\boldsymbol{\alpha}_{-ij}, \tilde{\alpha}_{ij})$ such that $\tilde{\alpha}_{ij} \in \mathcal{V}$ coincide on all arcs except possibly (i, j). This allows us to include the case where α_{ij} is such that $t_{ij} = 0$. Note that, for an increase in $\tilde{\alpha}_{ij} > \alpha_{ij}$ to be effective, it must be the case that $\tilde{t}_{ij} > 0$. Using the same method as in the proof of Theorem 3, we can write the consumption of any agent $k \in S_i$ as an increasing function of i's consumption, $g_k(\tilde{y}_i)$, and of any agent $\ell \in S_j$ as an increasing function of j's consumption, $g_\ell(\tilde{y}_j)$. Let $G(\tilde{y}_i) = \sum_{k \in S_i} g_k(\tilde{y}_i)$ and $H(\tilde{y}_j) = \sum_{\ell \in S_j} g_\ell(\tilde{y}_j)$. In the equilibrium transfer network, no money flows in or out of $S_i \cup S_j$, therefore

$$G(\tilde{y}_i) + H(\tilde{y}_j) = y^0(S_i \cup S_j) = G(y_i) + H(y_j). \tag{7}$$

Now, note that, in the initial network $\boldsymbol{\alpha}$, we must have $u'_i(y_i) \geq \alpha_{ij} u'_j(y_j)$. Since the increase is effective, $\tilde{t}_{ij} > 0$ and hence $u'_i(\tilde{y}_i) = \tilde{\alpha}_{ij} u'_j(\tilde{y}_j)$. Let $h(\alpha, x) = \big(u'_i\big)^{-1}\big(\alpha u'_j(x)\big)$. It is increasing in x and decreasing in α, and we can write $\tilde{y}_i = h(\tilde{\alpha}_{ij}, \tilde{y}_j)$ and $y_i \geq h(\alpha_{ij}, y_j)$. Replacing in (7), we have

$$G\big(h(\tilde{\alpha}_{ij}, \tilde{y}_j)\big) + H(\tilde{y}_j) \geq G\big(h(\alpha_{ij}, y_j)\big) + H(y_j) > G\big(h(\tilde{\alpha}_{ij}, y_j)\big) + H(y_j),$$

where the second inequality comes from $\tilde{\alpha}_{ij} > \alpha_{ij}$. Since $G\big(h(\tilde{\alpha}_{ij}, \cdot)\big) + H(\cdot)$ is an increasing function, this implies $\tilde{y}_j > y_j$, which immediately implies that $\tilde{y}_\ell > y_\ell$ for every $\ell \in S_j$. We show similarly that consumption of every agent in S_i decreases strictly.

Next, consider an effective increase $\tilde{\alpha}_{ij} > \alpha_{ij}$. Let $U = \{k : \tilde{y}_k > y_k\} \neq \varnothing$. If $k \in U$ and $t_{kl} > 0$, then $\alpha_{kl} u'_l(y_l) = u'_k(y_k) > u'_k(\tilde{y}_k) \geq \tilde{\alpha}_{kl} u'_l(\tilde{y}_l)$ and hence $l \in U$. Winners give to winners in $\mathbf{T}$. Similarly if $\tilde{t}_{lk} > 0$ and $(l, k) \neq (i, j)$, then $l \in U$. Winners receive from winners in $\tilde{\mathbf{T}}_{-ij}$. If $i \in U$ or $j \notin U$ or $\tilde{t}_{ij} = 0$, then $y^0(U) \leq y(U) < \tilde{y}(U) \leq y^0(U)$ which is impossible. Thus, $\tilde{t}_{ij} > 0$ and j is a winner. Any $k \in S_j(\mathbf{T} \cap \tilde{\mathbf{T}})$ can be reached from j through a path in $\mathbf{T} \cap \tilde{\mathbf{T}}_{-ij}$, with links flowing downards in $\mathbf{T}$ and upwards in $\tilde{\mathbf{T}}_{-ij}$. Hence k is also a winner. A similar argument applies to i and losers. $\square$

REFERENCES

Acemoglu, Daron, Garcia-Jimeno, Camilo and James A. Robinson. 2015. "State Capacity and Economic Development: A Network Approach." *American Economic Review* 105(8): 2364-2409.

Ahuja, Ravindra K., Magnanti, Thomas L., and James B. Orlin. 1993. *Network Flows : Theory, Algorithms, and Applications.* Prentice Hall, Englewood Cliffs, N.J.

Alesina, Alberto and George-Marios Angeletos. 2005. "Fairness and Redistribution." *American Economic Review* 95(4): 960-980.

Alger, Ingela and Jörgen W. Weibull. 2010. "Kinship, Incentives, and Evolution." *American Economic Review* 100(4): 1727-1760.

Allouch, Nizar. 2015. "On the Private Provision of Public Goods on Networks." *Journal of Economic Theory* 157: 527-552.

Altig, David and Steven J. Davis. 1992. "The Timing of Intergenerational Transfers, Tax Policy, and Aggregate Savings." *American Economic Review* 82(5): 1199-1220.

Ambrus, Attila, Mobius, Markus and Adam Szeidl. 2014. "Consumption Risk-Sharing in Social Networks." *American Economic Review* 104(1): 149-182.

Andreoni, James. 1989. "Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence." *Journal of Political Economy* 97(6): 1447-1458.

Angelucci, Manuela and Giacomo De Giorgi. 2009. "Indirect Effects of an Aid Program: How Do Cash Injections Affect Non-Eligibles' Consumption?" *American Economic Review* 99(1): 486-508.

Angelucci, Manuela, De Giorgi, Giacomo and Imran Rasul. 2012. "Resource Pooling within Family Networks: Insurance and Investment." working paper *Stanford University.*

Arrow, Kenneth. 1981. "Optimal and Voluntary Income Distribution." in Steven Rosefielde ed., *Economic Welfare and the Economics of Soviet Socialism: Essays in Honor of Abram Bergson*, Cambridge University Press.

Barro, Robert J. 1974. "Are Government Bonds Net Wealth?" *Journal of Political Economy* 82(6): 1095-1117.

Becker, Gary S. 1974. "A Theory of Social Interactions." *Journal of Political Economy* 82(6): 1063-1093.

Bergstrom, Theodore C. 1999. "Systems of Benevolent Utility Functions." *Journal of Public Economic Theory* 1(1): 71-100.

Bergstrom, Theodore C., Blume, Lawrence and Hal Varian. 1986. "On the Private Provision of Public Goods." *Journal of Public Economics* 29: 25-49.

Bernheim, B. Douglas. 1986. "On the Voluntary and Involuntary Provision of Public Goods" *American Economic Review* 76(4): 789-793.

Bernheim, B. Douglas and Kyle Bagwell. 1988. "Is Everything Neutral?" *Journal of Political Economy* 96(2): 308-338.

Bernheim, B. Douglas and Oded Stark. 1988. "Altruism within the Family Reconsidered: Do Nice Guys Finish Last?" *American Economic Review* 78(5): 1034-1045.

Bertsekas, Dimitri P. 2015. *Convex Optimization Algorithms*. Athena Scientific.

Bloch, Francis, Genicot, Garance and Debraj Ray. 2008. "Informal Insurance in Social Networks." *Journal of Economic Theory* 143: 36-58

Bramoullé, Yann and Rachel Kranton. 2007a. "Public Goods in Networks." *Journal of Economic Theory* 135(1): 478-494.

Bramoullé, Yann and Rachel Kranton. 2007b. "Risk Sharing Networks." *Journal of Economic Behavior and Organization* 64: 275-294.

Bramoullé, Yann, Rachel Kranton and Martin D'Amours. 2014. "Strategic Interaction and Networks." *American Economic Review* 104(3): 898-930.

Bruce, Neil and Michael Waldman. 1991. "Transfers in Kind: Why They Can Be Efficient and Nonpaternalistic." *American Economic Review* 81(5): 1345-51.

Cornes, Richard and Jun-Ichi Itaya. 2010. "On the Private Provision of Two or More Public Goods." *Journal of Public Economic Theory* 12(2): 363-385.

Cox, Donald. 1987. "Motives for Private Income Transfers." *Journal of Political Economy* 95(3): 508–546.

Cox, Donald, Hansen, Bruce E. and Emmanuel Jimenez. 2004. "How Responsive are Private Transfers to Income? Evidence from a Laissez-Faire Economy." *Journal of Public Economics* 88: 2193-2219.

De Weerdt, Joachim and Stephan Dercon. 2006. "Risk Sharing Networks and Insurance against Illness." *Journal of Development Economics* 81: 337-356.

De Weerdt, Joachim and Marcel Fafchamps. 2011. "Social Identity and the Formation of Health Insurance Networks." *Journal of Development Studies* 47(8): 1152-1177.

Fafchamps, Marcel and Flore Gubert. 2007. "The Formation of Risk Sharing Networks." *Journal of Development Economics* 83(2): 326-350.

Fafchamps, Marcel and Susan Lund. 2003. "Risk Sharing Networks in Rural Philippines."

Journal of Development Economics 71: 261-287.

Foster, Andrew D. and Mark R. Rosenzweig. 2001. "Imperfect Commitment, Altruism, and the Family: Evidence from Transfer Behavior in Low-Income Rural Areas." *Review of Economics and Statistics* 83(3): 380–407.

Franke, Jörg and Tahir Öztürk. 2015. "Conflict Networks." *Journal of Public Economics* 126: 104-113.

Galichon, Alfred. 2016. *Optimal Transport Methods in Economics.* Princeton: Princeton University Press.

Goyal, Sanjeev, Konovalov, Alexander and José Luis Moraga-González. 2008. "Hybrid R&D." *Journal of the European Economic Association* 6(6): 1309–1338.

Hori, Hajime and Sadao Kanaya. 1989. "Utility Functionals with Nonpaternalistic Intergenerational Altruism" *Journal of Economic Theory* 49: 241-265.

Laitner, John. 1991. "Modeling Marital Connections among Family Lines." *Journal of Political Economy* 99(6): 1123–1141.

Laitner, John. 1998. "Bequests, Gifts, and Social Security." *Review of Economic Studies* 55(2): 275-299.

Lee, Ronald, Donehove, Gretchen and Tim Miller. 2011. NTA Country Report, US, 2003. *National Transfer Accounts.* URL: http://www.ntaccounts.org.

Leider, Stephen, Mobius, Markus M., Rosenblat, Tanya and Quoc-Anh Do. 2009. "Directed Altruism and Enforced Reciprocity in Social Networks." *Quarterly Journal of Economics* 124(4): 1815-1851.

Ligon, Ethan and Laura Schechter. 2012. "Motives for Sharing in Social Networks." *Journal of Development Economics* 99: 13-26.

Mercier Ythier, Jean. 2006. "The Economic Theory of Gift-Giving: Perfect Substituability of Transfers and Redistribution of Wealth." in *Handbook of the Economics of Giving, Altruism and Reciprocity*, Kolm and Mercier Ythier eds., North Holland.

Pollak, Robert A. 2003. "Gary Becker's Contributions to Family and Household Economics." *Review of Economics of the Household* 1: 111-141.

Pollak, Robert A. 1988. "Tied Transfers and Paternalistic Preferences." *American Economic Review* 78(2): 240-244.

Voorneveld, Mark. 2000. "Best-Response Potential Games." *Economics Letters* 66: 289-295.

Worldbank. 2011. *Migration and Remittances Factbook 2011, 2*[d] *edition.* Washington DC.