Political Competition 2

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Limits of The Downsian Model

• The candidates are perfectly informed about the preferences of the voters (or at least they know the preferred policy of the median voter).

• The framework is essentially limited to one-dimensional policy spaces.

• Political parties are exogenously given and candidates do not self-select.

• Candidates can commit to applying their campaign promises.

• Platforms are non-ambiguous.
Outline

1. Probabilistic Voting: Random Utility Model
   a. Office seeking candidates
   b. Ideological Candidates
2. Probabilistic Voting: Idiosyncratic Biases
3. Probabilistic Voting and Redistribution
4. Citizen Candidate Models
Random Utility Model

• Suppose the preferences of the voters satisfy either
  (i) SP
  (ii) SCP and the median voter’s preferences are SP.

• Then majority voting produces a single-peaked ordering $\succeq^{mv}$ on $\Pi$. Let $\pi_m$ be the maximal element of this ordering (the favorite policy of the median voter).

• To capture uncertainty about preferences, we assume that the candidates do not observe $\pi_m$ but instead believe that $\pi_m \sim F(.)$. We will show that this is enough to capture all the relevant uncertainty about $\succeq^{mv}$.

• We assume that the beliefs of the candidates coincide: polls are public information.
Double Median

• Let $\pi_m^* = F^{-1}(1/2)$ be the median (relative to the uncertainty) favorite policy of the median voter.

**Theorem (Convergence)**

*If the two candidates are office seeking, there exists a unique pure strategy Nash equilibrium in which both candidates choose the same platform $\pi_m^*$.***
Proof 1

• For simplicity, we assume that $F(.)$ has full support.

• Suppose the candidates announce $\pi_L < \pi_R$.

• $\pi_L (\pi_R)$ wins for all the profiles with $\pi_m < \pi_L (\pi_m > \pi_R)$

• Look at the deviation $\pi_L \rightarrow \pi'_L = \pi_L + \frac{2}{3}(\pi_R - \pi_L)$

• Then for all the profiles with $\pi_m < \pi'_L$, $\pi'_L$ is a winner
Proof 2

• Two cases

  (i) If for some of these profiles $\pi_L$ was losing, then the deviation is profitable.

  (ii) If $\pi_L$ was already winning for all of these profiles, then $\pi_R \rightarrow \pi'_R = \pi_R - \frac{2}{3}(\pi_R - \pi_L)$ is a profitable deviation from $\pi_R$.

• Hence $\pi^L = \pi^R$.

• But then if $\pi_L = \pi_R \neq \pi^*_m$, any candidate can move to $\pi = \frac{\pi_L + \pi^*_m}{2}$ and win with a probability greater than $F(\pi) > 1/2$. 
Ideological Candidates: an Example

• Π is an interval of \( \mathbb{R} \) that contains \([-1, 1]\).

• All voters have symmetric single-peaked preferences

• \( \pi_m \) is uniformly distributed on \([-\delta, \delta]\).

• Candidates \( \ell \) and \( r \) with preferences

\[
\begin{align*}
  u_\ell(\pi) &= -(\pi + 1)^2, \\
  u_r(\pi) &= -(\pi - 1)^2
\end{align*}
\]

• \( \ell \) wins if \( \pi_m < \frac{\pi_\ell + \pi_r}{2} \)
Equilibrium: Partial Convergence

• In equilibrium, it must be true that \( \frac{\pi_\ell + \pi_r}{2} \in [-\delta, \delta] \).

• By symmetry \( \pi^{*}_\ell = -\pi^{*}_r \).

• \( \ell \)'s payoff

\[
-\left(\pi_\ell + 1\right)^2 \frac{1}{2\delta} \left(\frac{\pi_\ell + \pi_r}{2} + \delta\right) - \left(\pi_r + 1\right)^2 \frac{1}{2\delta} \left(\delta - \frac{\pi_\ell + \pi_r}{2}\right)
\]

• Then by taking the FOC: \( \pi^{*}_\ell = -\frac{\delta}{1+\delta} = -\pi^{*}_r \).

• Hence there is partial convergence, and \( \pi^{*}_\ell, \pi^{*}_r \to \pi_m = 0 \) as \( \delta \to 0 \).
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Probabilistic Voting


• The policy space $\Pi \subseteq \mathbb{R}^n$.

• There are two exogenously given, office seeking candidates $A$ and $B$ who choose policies $\pi_A$ and $\pi_B$ to maximize their vote share.
  
  • For example, because a higher share of the votes leads to more post-election power (whether they win or they lose), or more opportunities to earn rents, or increased campaign funding in future elections.

• Campaign promises are binding.
Voters Preferences

• The preferences of voter $i$ over $\Pi$ are given by $u_i(\pi)$.

• Voters also care about other dimensions that the candidates cannot control (image, personality, likeability, perceived policy positions on issues that are left out of the debate...)

• The payoff of $i$ if candidate $k$ is elected is given by

$$U_i(k) = u_i(\pi_k) + \varepsilon_i^k$$

• The likeability factors $\varepsilon_i^k$ are random from the point of view of the candidates

$$\varepsilon_i^B - \varepsilon_i^A \sim F_i(.)$$
Vote Shares

• Regularity Assumptions: $u_i(.)$ and $F_i(.)$ continuously differentiable, the density of $F_i(.)$ is $f_i(.)$.

$$\Pr(i \text{ votes for } A) = \Pr(U_i(A) - U_i(B) > 0) = F_i(u_i(\pi_A) - u_i(\pi_B))$$

• Candidate A’s expected number of votes is:

$$V_A(\pi_A, \pi_B) = \sum_{i \in I} F_i(u_i(\pi_A) - u_i(\pi_B))$$

• B’s expected number of votes is:

$$V_B(\pi_A, \pi_B) = \#I - \sum_{i \in I} F_i(u_i(\pi_A) - u_i(\pi_B))$$
Convergence

• Assumptions (see Banks and Duggan, 2006)
  1. Π is compact and convex
  2. \( u_i(.) \) is concave.
  3. \( V_A(\pi_A, \pi_B) \) is strictly concave in \( \pi_A \) and strictly convex in \( \pi_B \).

• Under these assumptions, there exists a unique Nash equilibrium in which both candidates choose the same platform \( \pi^* \).

• Furthermore \( \pi^* \) maximizes the following weighted social welfare function

\[
\pi^* = \arg\max_{\pi} \sum_i f_i(0) u_i(\pi),
\]

in which \( i \)'s weight is \( f_i(0) \).
Partial Argument

• In a Nash equilibrium

\[ \pi_A^* \in \arg \max_{\pi} \sum_{i \in I} F_i \left( u_i(\pi) - u_i(\pi_B^*) \right) , \]

\[ \pi_B^* \in \arg \max_{\pi} - \sum_{i \in I} F_i \left( u_i(\pi_A^*) - u_i(\pi) \right) . \]

• First Order Conditions for a Nash equilibrium with convergence

\( (\pi_A^* = \pi_B^* = \pi^*) \)

\[ \sum_{i \in I} f_i(0)u_i'(\pi^*) = 0. \]

• This condition is the same as the FOC for the program

\[ \max_{\pi} \sum_{i} f_i(0)u_i(\pi) \]
Interpreting the Result

• Suppose a voter’s utility is given by

\[ U_i(C) = u_i(\pi C) + \alpha_i \varepsilon^C, \]

where

(i) \( \varepsilon^C \sim F(.) \) common to all voters.

(ii) \( \alpha_i > 0 \) is the weight of the bias for voter \( i \).

• Then the weight of voter \( i \) is \( f(0) \frac{\alpha_i}{\alpha_i} \).

• Intuition: the voters who care more about policy and less about the idiosyncratic factor have more weight in the political outcome.

• Note that the political outcome is ex ante Pareto efficient whenever \( E [\varepsilon_i^A - \varepsilon_i^B] = 0 \).

• It is also Pareto efficient if we decide that idiosyncratic biases should be ignored.
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Probabilistic Voting and Redistribution

• Lindbeck and Weibull (1987)

• Voters $\mathcal{I} = \{1, \ldots, l\}$, with incomes $y_i > 0$.

• A redistribution policy $t \in \mathbb{R}^l$ is feasible and budget balanced if it satisfies:

  (i) $y_i + t_i > 0$

  (ii) $\sum_{i \in \mathcal{I}} t_i = 0$

• Preferences with $v'_i(.) > 0$ and $v''_i(.) < 0$ and $v'_i(0) = \infty$

  $$u_i(t, C) = v_i(y_i + t_i) + \varepsilon_i^C$$

• $\varepsilon_i^B - \varepsilon_i^A \sim F_i(.)$
Equilibrium

• Nash equilibrium

\[ t^A \in \arg \max_t \sum_{i \in \mathcal{I}} F_i \left( v_i(y_i + t_i) - v_i(y_i + t_i^B) \right) \quad \text{s.t.} \quad \sum_i t_i = 0, \]

\[ t^B \in \arg \max_t \sum_{i \in \mathcal{I}} -F_i \left( v_i(y_i + t_i^A) - v_i(y_i + t_i) \right) \quad \text{s.t.} \quad \sum_i t_i = 0, \]

• Let \( \lambda^A \) and \( \lambda^B \) be the Lagrange multipliers associated with the constraint in each program. Then for every \( i \) and every \( C \in \{A, B\} \)

\[ v'_i(y_i + t_i^C)f_i \left( v_i(y_i + t_i^A) - v_i(y_i + t_i^B) \right) = \lambda^C. \]
Policy Convergence

• Hence the following ratio is constant across individuals $i$

$$\frac{v_i'(y_i + t^A_i)}{v_i'(y_i + t^B_i)}$$

• Suppose $t^A \neq t^B$, then there exists $j$ such that $t^A_j > t^B_j$.

• But then it is true for every $i$ that $t^A_i > t^B_i$.

• But then $\sum_i t^A_i > \sum_i t^B_i$

• $t^A$ and $t^B$ cannot both be budget balanced!
Conclusions

- Hence in any Nash equilibrium in pure strategies, there is policy convergence $t^A = t^B = t^*$.

- Then there exists a constant $\lambda > 0$ such that
  $$\sum_{i \in I} v'_i(y_i + t^*_i)f_i(0) = \lambda.$$  

- And
  $$t^* = \arg \max_t \sum_i f_i(0)v_i(y_i + t) \text{ s.t. } \sum_i t_i = 0.$$  

- The voters who receive higher transfers are those with higher $f_i(0)$. As before, citizens who care less about the idiosyncratic factor receive more.

- Note that the cause of policy convergence is different than in the previous probabilistic voting framework.
Remarks on Probabilistic Voting

• The clear advantage of probabilistic voting models is that they allow to deal with multi-dimensional policy space.

• They also introduce some uncertainty in elections which seems realistic.

• The electoral game always admits a mixed strategy equilibrium.

• However, to ensure existence of a pure strategy equilibrium (or uniqueness, or policy convergence), we need to make assumptions that imply that the amount of uncertainty/randomness is substantial. Put differently, voters must care relatively little about the policy in question.
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Citizen Candidates

- Besley and Coate (1997), Osborne and Slivinski (1996)

- Idea: candidates are citizens who decide to step up to defend their ideal policy.

- Candidacy is endogenous.

- There is no commitment.
Model

- $\mathcal{I} = \{1, \ldots, I\}$

- $\Pi_i$ is the set of policies available to $i$ (citizens may have different policy-making competence).

  \[ \Pi = \bigcup_{i \in \mathcal{I}} \Pi_i \]

- Utility $U_i(\pi, j)$, where $j \in \mathcal{I} \cup \{0\}$ (ego-rents, likeability...)

- Every citizen can decide to run at cost $\delta$.

- Elections: the candidate with the highest number of votes wins, (uniform probabilities in case of a tie).

- If no one runs, the default policy is $\pi_0 \in \bigcap_{i \in \mathcal{I}} \Pi_i$
Timing

1. Candidates declare themselves.

2. Citizens decide for whom to vote among the declared candidates.

3. The elected candidate makes a policy choice.

We analyze these choices in reverse order...
The citizen who wins implements her preferred policy, anything else is not credible (no commitment device)

\[ \pi_k^* = \arg \max_{\pi \in \Pi_k} U_k(\pi, k). \]

Hence we can associate a utility profile \((u_{1k}, \ldots, u_{Ik})\) to each candidate \(k\) (and to the no candidate situation \(k = 0\)), with

\[ u_{jk} = U_j(\pi_k^*, k) \]
Voting

• Let $\mathcal{C} \subset \mathcal{I}$ be the set of declared candidates.

• Citizens can vote for a candidate in $\mathcal{C}$ or abstain, $\nu_i \in \mathcal{C} \cup \{0\}$.

• A voting profile $\nu = (\nu_1, \cdots, \nu_I)$.

• Let $\mathcal{W}(\mathcal{C}, \nu)$ be the set of winning candidates under $\nu$.

• Then the probability that $i$ wins the election is

$$P_i(\mathcal{C}, \nu) = \begin{cases} 1/\#\mathcal{W}(\mathcal{C}, \nu) & \text{if } i \in \mathcal{W}(\mathcal{C}, \nu) \\ 0 & \text{otherwise} \end{cases}$$
In a voting equilibrium, every voter correctly anticipates what policy candidate $k$ will choose if she wins.

$\nu^*$ is a voting equilibrium if (i) and (ii) hold

(i) $\nu^*_i \in \arg\max_{\nu_i \in C \cup \{0\}} \sum_{k \in C} P_k(C, (\nu_i, \nu^*_{-i})) u_{ik}$

(ii) $\nu^*_i$ is not a weakly dominated voting strategy.

Note: Ruling out weakly dominated strategies implies sincere voting in two-candidate elections.

A voting equilibrium exists for any nonempty candidate set. With more than 3 candidates there are multiple equilibria in general.
Entry

- Citizen $i$'s (pure) entry strategy is $e_i \in \{0, 1\}$.
- $e = (e_1, \cdots, e_l)$, $C(e) = \{i | e_i = 1\}$.
- Suppose $\nu(C)$ is the commonly anticipated voting strategy when the set of candidates is $C$.
- Given $\nu(.)$ and $e$, the expected payoff of $i$ is

$$U_i(e, \nu(.)) = \sum_{j \in C(e)} P_i(C(e), \nu(C(e))) u_{ij} + P_0(C(e)) u_{i0}$$

where $P_0(\emptyset) = 1$ and $P_0(C) = 0$ otherwise.
Existence

We allow for mixed strategy at the entry stage: citizen $i$ chooses to enter with probability $\gamma_i$.

**Theorem (Besley and Coate, 1997)**

*There exists a political equilibrium* $\{\gamma, \nu(.)\}$

- Typically, there are many equilibria, with one, two or more candidates.
An Example: Public Good Provision

- \( \mathcal{I} = [0, 1] \)
- Income \( y_i \sim F(\cdot) \)
- \( y_m = F^{-1}(1/2) < \bar{y} = \int ydF(y) \)
- Preferences: \( u_i(c_i, g) = c_i + H(g) \) with \( H' > 0, H'' < 0 \)
- Balanced Budget: \( g = \tau \bar{y} \).
- Hence \( U_i(g) = (1 - g/\bar{y}) y_i + H(g) \)
Voters’ ideal policies

• \( g_i^* = H^{-1} \left( \frac{Y_i}{Y} \right) \)

• With Downsian electoral competition the political outcome is \( g_m^* \).

• What happens with citizen candidates?

• If a citizen runs, she cannot commit to apply anything else than \( g_i^* \).

• Let \( \delta \) be the entry cost, and \( \hat{g} \) the status quo policy.
Median Voter

- \( U_i(g) = (1 - g/y) y_i + H(g) \) satisfies SCP (it has ID in \((-y_i, g))\).

- Hence the median voter’s preferences determine the outcome of binary electoral contests.

\[
U_m(g) > U_m(g') \Rightarrow g \succ^{mv} g'
\]

- With more than two competing policies, there is scope for strategic voting...
One-Candidate Equilibria

• If the median voter runs, no one can beat him, so no other candidate would enter.

• The median voter wants to run if

\[ U_m(g_m) - \delta > U_m(\hat{g}). \]

• Are there other one-candidate equilibria?
One Candidate Equilibria

\[ U_m(.) \]

\[ g_m \]
One Candidate Equilibria

$U_m(.)$

$g_m \succ m v g_i$

$g_i \succ g_m$

$\succ m v g_i$
One Candidate Equilibria

No Entry

$U_m(.)$

$g_m$

$g_i$

$\succ^m v g_i$
One Candidate Equilibria

\[ i \text{ wants to enter } U_m(.) \]

\[ g_m < g_i \succ mv g_i \]

\[ > \delta \]
Theorem

There exists an interval $Y$ of incomes with $y_m \in Y$ such that for every $i$ with $y_i \in Y$ there exists a one-candidate equilibrium in which $i$ is the candidate, and if $y_i \notin Y$, then there is no one-candidate equilibrium such that $i$ is the candidate.

- In any one-candidate equilibrium, the political outcome is “close to” the median.
Two-Candidate Equilibria

- Suppose $\ell$ and $r$ are both running.
- Then each of them must stand a chance to win, hence
  \[ U_m(g_\ell^*) = U_m(g_r^*) \]
- And each of them must prefer to run
  \[
  \frac{1}{2} \{ U_\ell(g_\ell^*) + U_\ell(g_r^*) \} - \delta \geq U_\ell(g_r^*) \Rightarrow U_\ell(g_\ell^*) - U_\ell(g_r^*) \geq 2\delta
  \]
  \[
  \frac{1}{2} \{ U_r(g_r^*) + U_r(g_\ell^*) \} - \delta \geq U_r(g_\ell^*) \Rightarrow U_r(g_r^*) - U_r(g_\ell^*) \geq 2\delta
  \]
- Hence $g_\ell^*$ and $g_r^*$ must be sufficiently far. Assume $g_\ell^* < g_r^*$
Two-Candidate Equilibria

• No other citizen should be willing to enter...

• Only citizens $C$ such that $g^*_\ell < g^*_C < g^*_r$ may have an incentive to enter (why?)

• The pivotal voters are $i$ and $j$ such that

$$U_i(g^*_C) = U_i(g^*_\ell),$$

$$U_j(g^*_C) = U_j(g^*_r).$$

• And $C$ gets $F(y_j) - F(y_i)$ votes.
Two-Candidate Equilibria

- $C$ wins if
  \[ F(y_j) - F(y_i) > \max\{F(y_i), 1 - F(y_j)\} \]  
  (1)

- And $C$ wants to run if
  \[ U_C(g_C^*) - \delta > \frac{1}{2} \{U_\ell(g_\ell^*) + U_\ell(g_r^*)\} \]  
  (2)

- Both (1) and (2) tend to be satisfied if $g_\ell^*$ and $g_r^*$ are not too far.
Remarks 1

• In both one and two-candidate equilibria, the outcome is no longer necessarily the preferred policy of the median voter.

• However, the median voter plays an important role in the characterization of these equilibria.

• In the two-candidate equilibria, a “left” and a “right” party emerge.

• In the one-candidate equilibrium, the status quo plays an important role.

• There are also three-candidate equilibria etc...
Remarks 2

- This model does not presume that commitment is possible.
- It is a possible approach towards endogenizing party formation.
- There is a small literature on the formation of political parties.
- However the multiplicity of equilibria can be a problem.
Topics

- Political Parties
- Ambiguity in Elections (strategic or involuntary)
- Applications of the citizen candidate model
- The political economy of taxation.
- Legislative Bargaining
- Lobbying
- Campaign Funding
- Elections with Imperfect Information