Outline

• CAPM and the Firm

• Debt, Leverage, and the Limits of CAPM

• Modigliani-Miller: No Taxes

• Modigliani Miller: Corporate Taxes

• Modigliani-Miller in General Equilibrium
Financing Projects

• A firm invests in new projects in order to make profits.

• We saw that a good way to evaluate project was the NPV criterion:

  • For a risk-neutral firm:

\[
\sum_{t=0}^{T} \frac{EC_t}{(1 + r)^t}
\]

• Problem: what is the right interest rate for the firm?

• It is the rate at which a firm can procure fundings on the financial markets
There are two ways a firm can fund itself on the market:

- **Debt**: the firm can issue corporate bonds.
- **Stock**: the firm can issue new equity.

The cost of capital of the firm is a weighted average of the cost of debt and the cost of equity.

- CAPM gives us a way to price the cost of capital.
- This makes CAPM a foundation for corporate finance.
Example

- United Semi Conductor considers a new project to increase its size.
- Suppose that the firm is 100% equity financed and its stock has a $\beta = 1.3$.
- Suppose also that $r^f = 5\%$ and the market risk premium is $8\%$.
- By CAPM the cost of capital is given by (now we work with expected returns):
  \[ r = 5\% + 1.3 \times 8\% = 15.4\% \]
- This is the discount rate that the firm should used to evaluate the project: if it can repay $100$ next year, it can borrow $100/1.154 = 86.6$ today.
- We can evaluate the NPV at $15.4\%$ and reject any project with NPV $< 0$ or use IRR and accept any project with IRR above $15.4\%$. 
Remarks about Betas

- The betas come from covariations with business fluctuations.
- The sources of variations in betas can be subtle.
- **Example:** Consider two methods of production for the same product, sold at price $10:
  - Method 1: fixed cost $1,000/\text{year}$, variable cost $8/\text{unit}$.
  - Method 2: fixed cost $2,000/\text{year}$, variable cost $6/\text{unit}$.

- Method 2 has a higher margin $4 > 2$.
- Hence it is more responsive to sales and will have a higher $\beta$ (more risk). It will also be more expensive to fund.
Suppose you have \( A \) invested in an asset with rate of return \( r_A \) and \( B \) invested in an asset \( r_B \).

Then what is your combined return \( r_{A+B} \)?

\[
r_{A+B} = \frac{A}{A+B} r_A + \frac{B}{A+B} r_B
\]

Then, by bi-linearity of the covariance, your combined beta is:

\[
\beta_{A+B} = \frac{A}{A+B} \beta_A + \frac{B}{A+B} \beta_B
\]
Outline

- CAPM and the Firm
- Debt, Leverage and the Limits of CAPM
- Modigliani-Miller: No Taxes
- Modigliani Miller: Corporate Taxes
- Modigliani-Miller in General Equilibrium
Leverage

- Leverage is the use of debt to fund a project.
- We know that leverage increases risk.
- We want to understand how it affects corporate decisions.
- How leveraged should firms be?
Leverage and Equity

- Let $B$ denote the amount of debt and $S$ the amount of stock or equity.

\[
\beta_{asset} = \frac{S}{S+B}\beta_S + \frac{B}{S+B}\beta_B
\]

- Corporate debt generally has a low beta.
- Generally, assume $\beta_B = 0$ (but need to be careful).
- Hence the stock is more risky than the asset:

\[
\beta_S = \left(1 + \frac{B}{S}\right)\beta_{asset} > \beta_{asset}
\]

- Leverage increases the risk of the stock.
- The increase is proportional to the Debt/Equity ratio.
Cost of Capital

• Back to the cost of capital for a project financed by debt and equity:

\[ r_{WACC} = \frac{S}{S+B}r_S + \frac{B}{S+B}r_B \]

• WACC: Weighted Average Cost of Capital

• In a world with taxes, the formula is different since interest costs are deductible from corporate taxes:

\[ r_{WACC} = \frac{S}{S+B}r_S + \frac{B}{S+B}r_B (1 - T_C) \]

Cost of Debt After Tax

• Then for any project:

\[ NPV = C_0 + \sum_{t=1}^{T} \frac{EC_t}{(1 + r_{WACC})^t} \]
CAPM and Corporate Debt

• Assuming $\beta_B \neq 0$, the CAPM gives a symmetric view of debt and equity:

$$r_S = r^f + \beta_S(r^M - r^f)$$
$$r_B = r^f + \beta_B(r^M - r^f)$$

• And then we obtain that:

$$r_{WACC} = r^f + \beta_{asset}(r^M - r^f)$$

• Problem:
  • $\beta_B$ is generally small (empirical).
  • Then by CAPM: $r_B - r^f$ is small.

• This is rejected by the data: the risk premium on corporate bonds can be very high.
Corporate Bond Pricing

• The dominant source of risk on corporate bonds is default.

• It involves moral hazard so it is difficult to insure against (CDS are supposed to help).

• The markets cannot diversify away all the risks of default.

• Default has a big impact: the firm is effectively put out of the market.

• Bankruptcy is generally costly: it is complicated who is entitled to which remaining assets of the firm, some assets have to be sold at a low price through fire sales.
Corporate Bond Pricing

• This does not fit very well in the CAPM theory.

• To avoid this problem we will when possible:

  (i) Assume that $\beta_B = 0$.

  (ii) Recognize that the CAPM equation $r_B = r^f + \beta_B(r^M - r^F)$ has little value.

  (iii) Instead, in most applications, take the cost of debt $r_B$ to be exogenously given (and hence independent from the choices of the firm).

• But sometimes also work within the CAPM framework while acknowledging its limits.
Outline

• CAPM and the Firm
• Debt, Leverage, and the Limits of CAPM
• Modigliani-Miller: No Taxes
• Modigliani Miller: Corporate Taxes
• Modigliani-Miller in General Equilibrium
Value of the Firm

- The capital structure of the firm is its mix of debt and equity.
- Value of the Firm = Value of Debt + Value of Equity
- **Question**: Is it possible to change the value of the firm by changing its capital structure?
- Some applications:
  - Taking debt to pay dividends.
  - Using leverage to restructure a firm.
  - Using debt to break up a firm (LBO).
  - Mergers and acquisitions.
Maximizing the Value of the Firm

- The debt is a fixed payment every period (unless there is a default).

- Hence at any given debt level, maximizing the value of the firm is the same as maximizing the value of equity.

- So managers maximize the value of the stock which is what stockholders want.

- Note: we ignore agency problems.
The Trade-Off

• Leverage increases expected return and risk linearly:

\[ \beta_S = \left(1 + \frac{B}{S}\right) \beta_0 \]

\[ E r_S - r^f = \left(1 + \frac{B}{S}\right) (E r_0 - r^f) \]

• Where \( \beta_0 \) and \( r_0 \) are for the corresponding all equity firm.
Cost of Capital

- For the all-equity firm, we have $r_{WACC} = r_0$.
- For a firm financed by debt and equity:
  
  $$r'_{WACC} = \frac{B}{B+S}r_B + \frac{S}{B+S}r_S$$

- CAPM implies: $r^S = r^f + (1 + \frac{B}{S}) (r_0 - r^f)$.
- Hence:
  
  $$r'_{WACC} = \frac{B}{B+S}r^f + \frac{S}{B+S} \left( r^f + \left(1 + \frac{B}{S}\right) \left( r_0 - r^f \right) \right) = r_0$$

- Note: we used all the CAPM equation including the suspicious ones.
Hence the cost of capital is the same for the all equity firm and the leveraged firm.

Their future cash flows are discounted by the same amount and the NPV formula implies that the two firms have the same value.

This is the first result of Modigliani-Miller: the capital structure has no effect on the value of the firm.

A shareholder is indifferent between different capital structures.
Intuition

• An investor who likes leverage can achieve it by borrowing on the market to buy stock.

• Therefore any change in the capital structure performed by the management can be outdone by the investors.

• The problem with CAPM is clear in the intuition: in practice the investor is unlikely to face the same cost of borrowing as the firm. In CAPM, any debt costs $r^f$.

• Note however that with a margin account the investor can actually get the same rate as the firm by selling short the corporate bond on the financial markets.
• Now we can write:

\[ r_0 = r_{WACC} = \frac{B}{B + S} r_B + \frac{S}{B + S} r_S \]

• Hence we have the second MM equation:

\[ r_S = r_0 + \frac{B}{S} (r_0 - r_B) \]

• The required equity return is linear in \( B/S \).
Outline

• CAPM and the Firm
• Debt, Leverage, and the Limits of CAPM
• Modigliani-Miller: No Taxes
• Modigliani Miller: Corporate Taxes
• Modigliani-Miller in General Equilibrium
Earnings

• **EBIT**: Earnings Before Interest and Taxes

• Payment of interests: $r_B B$

• **EAT**: Earnings After Taxes

\[
EAT = (EBIT - r_B B)(1 - T_C)
\]

• This is the cash flow that goes to stockholders and bondholders (payment of principal).

• Debt can be used to reduce the share of the profits that goes to the government.

• The tax shield is the reduction in taxes due to the presence of debt: $T_C r_B B$. 
Value of the Firm

- Assume permanent cash flows.

- Annual cashflow: $EBIT(1 - T_C)$

- Value of the all equity firm: $V_0 = \frac{EBIT(1 - T_C)}{r_0}$

- Value of the levered firm:

$$V_L = \frac{EBIT(1 - T_C)}{r_0} + \frac{T_C r_B B}{r_B} = V_0 + \underbrace{T_C B}_{NPV \ of \ tax \ shield}$$
Modigliani-Miller 2

- Expected cash flow:

\[ Sr_S + Br_B = V_0 r_0 + T_C r_B B \]

- Hence:

\[ r_S = \frac{V_0}{S} r_0 - (1 - T_C) \frac{B}{S} r_B \]

- And:

\[ V_L = V_0 + T_C B = S + B \]

- Hence:

\[ \frac{V_0}{S} = 1 + (1 - T_C) \frac{B}{S} \]
Cost of Capital

- Finally, we have a new version of MM-2 for the cost of equity:

\[ r_S = r_0 + (1 - T_C) \frac{B}{S}(r_0 - r_B) \]

- The weighted cost of capital for the firm is:

\[ r_{WACC} = \frac{S}{V_L} r_S + (1 - T_C) \frac{B}{V_L} r_B \]

- Note: there is no reason to limit leverage in this formulation.

- In practice, the limit is the risk of default.
Outline

• CAPM and the Firm

• Debt, Leverage, and the Limits of CAPM

• Modigliani-Miller: No Taxes

• Modigliani Miller: Corporate Taxes

• Modigliani-Miller in General Equilibrium
• CAPM is a bit restrictive, we show MM-1 more generally.

• \( j = 1, \cdots, J \) firms with uncertain profits \( X_j^j(\omega) \).

• Each firm issues stock with total value \( S^j \) and corporate bonds with total value \( B^j \).

• A risk-free bond with return \( R^f \).

• \( i = 1, \cdots, I \) individuals with non-financial income \( w_i \).

• \( S^i_j \) and \( B^i_j \) are the value of \( i \)'s holdings.

• The returns on the stock are given by (R): \( S^i_j R^i_S = X^i_j - B^i_j R^f \)
Result

• Assumptions:
  (i) Individuals and all firms can borrow at the same rate.
  (ii) No default.

• Proposition: If there exists a general equilibrium with each firm having a particular debt-equity ratio and a particular value, then there exists another GE with any firm having any different debt-equity ratio but with the value of all firms and the market rate of interest unchanged.
Proof

• The value of the firm satisfies:

\[ V^j = S^j + B^j \]

• The budget constraint of the agent in period 1 is given by:

\[ \sum_j S^j_i + B_i = \sum_j \alpha_i^j S^j + B_i = w_i \]

• Then her financial income in state \( \omega \) is given by:

\[ Y_i(\omega) = \sum_j S^j_i R^j_S(\omega) + R^f \left( w_i - \sum_j S^j_i \right) \]
Proof

• Now consider an alternative world in which firm 1 issues no bond.

• The new financial income is given by

\[
\hat{Y}_i(\omega) = \sum_j \hat{S}_j^i \hat{R}_j^i(\omega) + \hat{R}_f^i \left( w_i - \sum_j \hat{S}_j^i \right)
\]

• Suppose \( \hat{R}_f^i = R_f^i \) and \( \hat{S}_j^i = S_j^i \) for \( j \geq 2 \).

• Then by (R), for \( j \geq 2 \), \( \hat{R}_j^i = R_j^i \).
Proof

• Then individuals can rebuild the leverage of the initial situation by borrowing the additional amount \( \frac{B^1}{S^1} S^1_i \) so that

\[
\hat{B}_i = B_i + \frac{B^1}{S^1} S^1_i
\]

• With this additional cash, \( i \) can purchase more equity in firm 1:

\[
\hat{S}^1_i = S^1_i + \frac{B^1}{S^1} S^1_i = S^1_i \frac{V_1}{S^1}
\]
Proof

• Then her financial income is given by

\[ \hat{Y}_i(\omega) = \frac{\hat{S}_1}{\hat{S}_1} X^1(\omega) + \sum_{j \geq 2} S_i^j R^j(\omega) + R^f \left( w_i - \sum_{j \geq 2} S_i^j - \hat{S}_1 \right) \]

\[ = S_i^1 \frac{V_1}{S_1^1} \left( X^1(\omega) - \frac{B^1}{S_1^1} S_i^1 R^f \right) + \sum_{j \geq 2} S_i^j R^j(\omega) + R^f \left( w_i - \sum_{j \geq 1} S_i^j \right) \]

\[ \frac{S_i^1}{S_1^1} \left( X^1(\omega) - B^1 R^f \right) = S_i^1 R^1_S(\omega) \]

\[ = Y_i(\omega) \]
Proof

• Hence the individual has the same opportunity set as before, so if she was maximizing utility in the initial situation she still is.

• Markets clear by construction:

\[ \sum_i \hat{S}_i = \sum_i S_i \frac{V_i}{S_1} = V_1 \]

• And then it must clear for the bond as well.
Comments

• Note that in the proof, individuals may have different beliefs about the probability of states of the world tomorrow.

• They need only agree that there will not be any default: for every $\omega$

$$X^i(\omega) > R^f B^i$$

• However the no default assumption remains a serious problem in this more general approach.

• We did not assume complete markets.
Thanks!