Financial Economics
CAPM

Eduardo Perez-Richet

Ecole Polytechnique
Outline

• Statistics 101

• CAPM equations

• Beta Factor Models

• CAPM in Complete Markets

• Mean Variance Representation

• Consumption Based CAPM

• Risk decomposition in CAPM
Some Facts

• Consider a state space $\Omega = \{1, \cdots, S\}$ with a distribution $\pi$.

• A random variable is a vector $x \in \mathbb{R}^S$.

• Expectation: $E x = \sum_S \pi_s x_s$

• Covariance: $cov(x, y) = E((x - E x)(y - E y)) = E(xy) - E x E y$

• Then $\mathcal{E} \equiv \{ x - E x \mid x \in \mathbb{R}^S \} = \{ x \mid E x = 0 \}$ is a vector space.

• We can define a norm (distance) on $\mathcal{E}$ by:

$$\|x - y\|^2 = Var(x - y) = Cov(x - y, x - y)$$
• \((\mathcal{E}, \text{cov}(., .))\) is a Hilbert space.

**Theorem (Hilbert)**

If \(X \subseteq \mathcal{E}\) is closed and convex, and \(y \in \mathcal{E}\), then there exists a unique \(x^* \in X\) such that

\[
x^* = \arg \min_{x \in X} \|x - y\|^2 = \arg \min_{x \in C} \text{Var}(x - y)
\]

Then \(x^* \equiv \text{proj}(y|X)\) is the *projection* of \(y\) on \(X\), and we can write

\[
y = \lambda x^* + y_{\perp},
\]

where \(\lambda \geq 0\) and \(y_{\perp}\) is *orthogonal* to \(X\). That is for every \(x \in X\),

\[
\text{cov}(x, y_{\perp}) = 0
\]
Illustration
Correlation an Projections

• Correlation:

\[
\text{corr}(x, y) = \frac{\text{cov}(x, y)}{\sqrt{\text{Var}(x)\text{Var}(y)}} = \left\langle \frac{x}{\|x\|}, \frac{y}{\|y\|} \right\rangle
\]

• Then we can show that \(\text{proj}(y|X)\) is the vector in \(X\) with maximal correlation with \(y\):

\[
x^* = \arg \max_{x \in X} \text{corr}(x, y)
\]
Regression

• If $y$ and $x$ are two random variables, we can always write

\[ y = \gamma + \beta x + \varepsilon \]

• With $\text{cov}(x, \varepsilon) = 0$, $E(\varepsilon) = 0$, and:

\[ \beta = \frac{\text{cov}(x, y)}{\text{var}(x)}. \]

• Why?

• Project $y - Ey$ on $x - Ex$, then you have the decomposition:

\[ y - Ey = \beta (x - Ex) + \varepsilon \]

• With $\text{cov}(x, \varepsilon) = 0$, $E(\varepsilon) = 0$
Getting $\beta$

- Then write that:

$$\text{cov}(x, y) = E \left( (x - Ex)(y - Ey) \right)$$

$$= \beta \text{Var}(x) + \text{cov}(x - Ex, \varepsilon)$$

$$= \beta \text{Var}(x)$$

$$\Rightarrow \beta = \frac{\text{cov}(x, y)}{\text{var}(x)}$$

- And:

$$\gamma = Ey - \beta Ex$$
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Setup

- Financial market: \((q, D)\) with \(K\) risky assets.
- Portfolios \(\theta \in \mathbb{R}^k\) with contingent payoff \(\theta \cdot d(s)\) and price \(\theta \cdot q\).
- Portfolios are scalable, we can normalize to consider only portfolios with price 1. These are returns:
  \[
  R^\theta(s) = \frac{\theta \cdot d(s)}{\theta \cdot q}
  \]
- Then:
  \[
  q(R^\theta) = 1
  \]
- The space of returns is:
  \[
  \mathcal{R} = \left\{ r \in \mathbb{R}^S \mid r = R^\theta \text{ for some portfolio } \theta \right\}
  \]
Properties of Returns

• $\mathcal{R}$ is a closed and convex set:

  • $\alpha R^\theta + (1 - \alpha) R^{\theta'} = R^{\theta''}$ where $\theta'' = \alpha \frac{\theta}{q(\theta)} + (1 - \alpha) \frac{\theta'}{q(\theta')}$. 

  • Why? First: $q(\theta'') = \alpha \frac{q(\theta)}{q(\theta)} + (1 - \alpha) \frac{q(\theta')}{q(\theta')} = 1$

  • And second:

    $R^{\theta''} = \frac{1}{q(\theta'')} \left( \frac{\theta \cdot d}{q(\theta)} + (1 - \alpha) \frac{\theta' \cdot d}{q(\theta')} \right) = \alpha R^\theta + (1 - \alpha) R^{\theta'}$

• $R^\theta = R^{-\theta}$

• Finding the return which is maximally correlated to $x$ means finding at the same time the portfolio whose payoff is maximally correlated to $x$ and the portfolio whose payoff is minimally correlated to $x$:

  $$corr(\theta \cdot d, x) = sign(q(\theta)) \cdot corr(R^\theta, x)$$
Origins of CAPM

- Sharpe (1964), Lintner (1965)

- It is the first, most famous and widely used model in asset pricing.

- CAPM is sometimes presented as a non-consumption based model.

- But in fact it is the result of additional restrictions on the consumption based equilibrium model.

- There are different sets of assumptions that can generate the CAPM.
  - On the market: either complete markets or no non-financial income in the second period.
  - On preferences/returns: (i) quadratic preferences or (ii) CARA preferences with normal returns.
The CAPM also assumes a risk free bond with return $R^f$.

For every risky portfolio $\theta$, we can write:

$$ER^\theta - R^f = \beta_\theta \left( ER^M - R^f \right),$$

where $M$ is the “market portfolio”, and

$$\beta_\theta = \frac{\text{cov} \left( R^\theta, R^M \right)}{\text{Var} \left( R^M \right)} = \text{corr} \left( R^\theta, R^M \right) \sqrt{\frac{\text{Var} \left( R^\theta \right)}{\text{Var} \left( R^M \right)}}$$

Equivalently, the CAPM is a model with a linear discount factor in the market portfolio:

$$m = a + bR^M.$$
Market Portfolio

• The market portfolio is the portfolio that delivers aggregate wealth in each state of the world.

• Think of it as the portfolio that consists of all the firms in a country.

• Concretely it is well approximated by an index (CAC 40, S&P500).

• So the CAPM gives you the risk premium (or excess return) of any portfolio as a function of the risk premium of the market.

• The risk premium comes from covariance with the market portfolio (a combination of correlation and variance).

• \( \beta_\theta \) is the “right” equilibrium measure of risk.

• Can you think of an example of an asset with low (high) risk premium?
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Risk Premium and Discount Factor

• Suppose that there is no arbitrage so that there exists a discount factor $m$.

• Then $1 = E(mR^\theta) = \text{cov}(m, R^\theta) + EmER^\theta$, hence:

$$ER^\theta - \frac{1}{Em} = -\frac{\text{cov}(m, R^\theta)}{Em}$$  \hspace{1cm} (RP 1)

• Note that if there is a risk free asset, then $R^f = \frac{1}{Em}$

• The risk premium is higher for assets with a low covariance with the discount factor.

• With complete markets, $m = \frac{\beta u'(\bar{e}_{t+1})}{u'(\bar{e}_t)}$, so the risk premium is higher for an asset that is positively correlated with the change in aggregate wealth.
Projection

- Let $\mathcal{R}_0 = \{r - Er \mid r \in \mathcal{R}\}$ and let

$$R^* - ER^* = \text{proj} \left( m \mid \mathcal{R}_0 \right)$$

- Then $R^*$ is the return most correlated to $m$:

$$R^* = \arg \max_{r \in \mathcal{R}} \text{corr}(r, m)$$

- With complete markets: $\text{corr}(R^*, m) = 1$.

- In general, we have the decomposition: $m = \gamma + \beta_{m,R^*} R^* + m_{\perp}$ with $m_{\perp} \perp \mathcal{R}_0 \Rightarrow \text{cov}(R^\theta, m_{\perp}) = 0$ for all $\theta$. 
Beta Factor Asset Pricing Formula

• So with (RP 1) and the decomposition:

\[
\frac{ER^\theta - \frac{1}{Em}}{ER^* - \frac{1}{Em}} = \frac{cov (R^\theta, m)}{cov (R^*, m)} = \frac{cov (R^\theta, R^*)}{var (R^*)} = \beta^*_\theta
\]

• And assuming a risk free asset, we obtain the beta factor asset pricing formula:

\[
ER^\theta - R^f = \beta^*_\theta \left( ER^* - R^f \right) \quad \text{(RP } \beta)\]

• Note: \( R^* \) is the return of the portfolio most correlated with \( m \) as well as the return of the portfolio least correlated with \( m \).
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First Setup

• Consider an exchange economy with two periods $t$ and $t + 1$, with complete markets and quadratic preferences.

• Then we can use a representative agent with quadratic preferences:

$$u(x) = x - \frac{1}{2a}x^2$$

• Then the discount factor is given by

$$m = k - K \bar{e}_{t+1}$$

• By completeness, there is a market portfolio $\theta^M$ that replicates aggregate endowment.

• Then the portfolio that maximizes correlation with $m$ is $-\theta^M$: it is perfectly correlated.

• The corresponding return is $R^M = R^\theta^M = R^{-\theta^M}$
CAPM equations

• Then by applying the $\beta$ factor formula:

$$ER^\theta - R^f = \beta_\theta \left( ER^M - R^f \right)$$

• Where:

$$\beta_\theta = \frac{\text{cov}(R^\theta, R^M)}{\text{var}(R^M)}$$

• The discount factor is:

$$m = k - Kq(\theta^M) R^M$$

• Another way to write the pricing equation in this case:

$$q(\theta) = k E(d) - K E(d \bar{e}_{t+1}) = a E(d) - b \text{cov}(d, \bar{e}_{t+1})$$
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Combining Portfolios

We can represent every portfolio as a point in the mean-variance space.

- Consider \( \theta'' = \frac{\alpha}{q(\theta)} \theta + \frac{1-\alpha}{q(\theta')} \theta' \)
- Then

\[
ER^{\theta''} = \alpha ER^\theta + (1-\alpha) ER^{\theta'}
\]

- And:

\[
\text{var}\left( R^{\theta''} \right) = \alpha^2 \text{var}(R^\theta) + (1-\alpha)^2 R^{\theta'}
+ 2\alpha(1-\alpha) \sqrt{\text{var}(R^\theta) \text{var}(R^{\theta'})} \text{corr}(R^\theta, R^{\theta'})
\]
Combining Portfolios

We can represent every portfolio as a point in the mean-variance space.

- Consider $\theta'' = \frac{\alpha}{q(\theta)} \theta + \frac{1-\alpha}{q(\theta')} \theta'$
- Then
  $$ER^{\theta''} = \alpha ER^{\theta} + (1-\alpha) ER^{\theta'}$$
- And:
  $$\text{var} \left( R^{\theta''} \right) = \alpha^2 \text{var} \left( R^{\theta} \right) + (1-\alpha)^2 R^{\theta'}$$
  $$+ 2\alpha(1-\alpha) \sqrt{\text{var} \left( R^{\theta} \right) \text{var} \left( R^{\theta'} \right)} \text{corr} \left( R^{\theta}, R^{\theta'} \right)$$
Combining Portfolios

We can represent every portfolio as a point in the mean-variance space.

• Consider \( \theta'' = \frac{\alpha}{q(\theta)} \theta + \frac{1-\alpha}{q(\theta')} \theta' \)

• Then

\[
ER^{\theta''} = \alpha ER^\theta + (1-\alpha) ER^{\theta'}
\]

• And:

\[
\text{var}(R^{\theta''}) = \alpha^2 \text{var}(R^\theta) + (1-\alpha)^2 R^{\theta'} + 2\alpha(1-\alpha)\sqrt{\text{var}(R^\theta)\text{var}(R^{\theta'})} \rho
\]
Mean Variance Representation

- Let: $\mathcal{R}^{mv} = \left\{ \left( E(R), \sqrt{\text{var}(R)} \right) \mid R \in \mathcal{R} \right\}$

- Then $\mathcal{R}^{mv}$ is a convex set.

- If the risk free return is not in $\mathcal{R}$:
Combining with the Bond

- Suppose that there exists a risk free bond $R^f$.

- Consider the return $R_\alpha$ of investing $-\infty < \alpha < 1$ in the bond and $1 - \alpha$ in $R^\theta$.

- $E R_\alpha = \alpha R^f + (1 - \alpha) E R^\theta$

- $\text{var}(R_\alpha) = (1 - \alpha)^2 \text{var}(R^\theta)$
Combining with the Bond

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- Consider the return $R_{\alpha}$ of investing $-\infty < \alpha < 1$ in the bond and $1 - \alpha$ in $R^\theta$.

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- $\text{var}(R_{\alpha}) = (1 - \alpha)^2 \text{var}(R^\theta)$

- Shorting the bond to buy an asset is called leverage, this figure illustrates how it increases variance and return linearly.
Mean-Variance Preferences

- The set of attainable returns is the cone between the two lines.
- $R_{mv}$ is the mean-variance return.
- Every agent with a utility $U(E_R, \text{Var}(R))$ (increasing in $E_R$, decreasing in $\text{var}(R)$) picks a return on the market line: this is the two funds theorem.
- The less risk averse agent picks a leveraged portfolio, while the more risk averse agent picks a non-leveraged portfolio.
Suppose that the bond is in zero net supply and that the other securities are real assets in positive net supply $\theta^k$.

In equilibrium, the portfolios of the agent must sum to the market portfolio.

Since $R_{mv}$ is the only real portfolio return on the market line, it must be the return of the market portfolio $R^M$. 

Market Portfolio
Mean-Variance Utilities

• When do we have mean-variance preferences in returns?

• Clearly when the utility is quadratic, the consumer only cares about the first two moments of her consumption in the second period but what about returns?

• Suppose that the consumer has utility:

\[ u_t(c_t) + \delta E u_{t+1}(c_{t+1}), \]

• With: \( u_{t+1}(x) = x - \frac{1}{2a} x^2. \)
Rewriting the Budget Constraint

- Budget constraint:

\[ c_t = e_t - \theta \cdot q \]

\[ c_{t+1}(s) = e_{t+1}(s) + \theta \cdot d(s) \]

- We can rewrite the second line as:

\[ c_{t+1}(s) = e_{t+1}(s) + (e_t - c_t)\tilde{\theta} \cdot R(s), \]

- Where \( R(s) \) is the vector of returns of the \( K \) securities and \( \tilde{\theta} = \frac{1}{q_{\tilde{\theta}}}(\theta_1 q_1, \cdots, \theta_K q_K) \) is just a weight vector.

- Note that \( R = \{ r \in \mathbb{R}^S \mid r = \tilde{\theta} \cdot R \text{ and } \sum_k \tilde{\theta}^k = 1 \} \).
Program of the Agent

- We can write the program this way:

\[
\max_{c_t, \tilde{R}} u_t(c_t) + \delta E u_{t+1} (e_{t+1} + (e_t - c_t)\tilde{R}) \quad \text{s.t.} \quad c_t \geq 0, \tilde{R} \in \mathcal{R}
\]

- But then if the solution of the program is a pair \((c^*, R^*)\) and if \(e_{t+1}\) has no uncertainty, we can write that:

\[
R^* = \arg \max_{\tilde{R} \in \mathcal{R}} u_t(c^*) + \delta E u_{t+1} \left(e_{t+1} + (e_t - c^*)\tilde{R}\right) = \arg \max_{\tilde{R} \in \mathcal{R}} 2 (a - e_{t+1}) E\tilde{R} - (e_t - c^*) \left((E\tilde{R})^2 + \text{var}(\tilde{R})\right)
\]

- Hence \(R^*\) is an optimal mean-variance return whenever the second period income has no uncertainty.
Another case in which the agent has mean-variance preferences over consumption is if:

- The utility function is CARA: \( u(x) = -e^{-\lambda x} \).
- Consumption is normally distributed.

We can show that if the agent has a CARA utility in the second period and returns (or dividends) are all normally distributed then the optimal return is an optimal mean-variance return.

See the problem set for an analysis of this framework.
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CCAPM

- The CCAPM is a generalization of the CAPM to incomplete markets with positive non-financial income in the second period.

- The preferences of consumers are given by:

\[ u_i^t(c_i^t) + \delta E u_i^{t+1}(c_i^{t+1}) \]

- Where:

\[ u_i^{t+1}(x) = x - \frac{1}{2a_i}x^2 \]

- The market consists of \( K \) risky securities \( d^k \) with prices \( q^k \) and a risk free bond with price \( q^f \).

- The bond is in zero net supply, while other securities are real assets in positive net supply \( \theta^k \).
Investor Program

$$\max_{\theta^f_i, \theta^i} u^i_t(c^i_t) + \delta E u^i_{t+1}(c^i_{t+1})$$

s.t.
$$c^i_t = e^i_t - \theta^f_i q^f - \theta^i \cdot q$$
$$c^i_{t+1}(s) = e^i_{t+1}(s) + \theta^f_i + \theta^i \cdot d(s)$$

- FOC with respect to $\theta^f_i$:
$$q^f u^i_t'(c^i_t) = \delta E \left( 1 - \frac{1}{a_i} c^i_{t+1} \right)$$

- FOC with respect to $\theta^k_i$:
$$q^k u^i_t'(c^i_t) = \delta E \left\{ \left( 1 - \frac{1}{a_i} c^i_{t+1} \right) d^k \right\}$$
Pricing

• Taking the ratio:

\[ \frac{q^k}{q_f} E \left( a_i - c_{i+1}^i \right) = E \left\{ \left( a_i - c_{i+1}^i \right) d^k \right\} = a_i E d^k - E \left( c_{i+1}^i d^k \right) \]

\[ = E \left( a_i - c_{i+1}^i \right) E d^k - \text{cov} \left( c_{i+1}^i, d^k \right) \]

• Summing across agents, and using the market clearing condition:

\[ \frac{q^k}{q_f} E \left( \bar{a} - \bar{c}_{i+1} \right) = E \left( \bar{a} - \bar{c}_{i+1} \right) E d^k - \text{cov} \left( \bar{c}_{i+1}, d^k \right) \]

• Hence:

\[ q^k = q_f \left( E d^k - \frac{\text{cov} \left( \bar{c}_{i+1}, d^k \right)}{E \left( \bar{a} - \bar{c}_{i+1} \right)} \right) \]
CCAPM equations

• Hence we have the CCAPM pricing equation for any portfolio $\theta$:

$$q^\theta = q^f \left( Ed^\theta - \frac{\text{cov} \left( \bar{c}_{t+1}, d^\theta \right)}{E (\bar{a} - \bar{c}_{t+1})} \right)$$  \quad (\text{CCAPM})

• In terms of returns:

$$ER^\theta - R^f = \frac{\text{cov} \left( \bar{c}_{t+1}, R^\theta \right)}{E (\bar{a} - \bar{c}_{t+1})}$$

• Since we can write the same for the market portfolio $\bar{\theta}$:

$$ER^\theta - R^f = \frac{\text{cov} \left( \bar{c}_{t+1}, R^\theta \right)}{\text{cov} \left( \bar{c}_{t+1}, R^M \right)} \left( ER^M - R^f \right)$$  \quad (\text{CCAPM'})
CAPM in Incomplete Markets

- If all income is financial in the second period, so that $e_{t+1}^i = 0$, then:
  \[ \bar{c}_{t+1} = d^M = q^M R^M \]

- And we recover the CAPM pricing equation:
  \[ ER^\theta - R^f = \frac{cov (R^M, R^\theta)}{var (R^M)} \left( ER^M - R^f \right) \]

- This is the usual framework for the CAPM.

- Another case in which we recover the pricing equation of the CAPM is if aggregate non-financial income in the second period has no uncertainty.
Individual Demand

• To find individual demand note that:

\[
\frac{q^k}{q^f} E \left( a_i - c^i_{t+1} \right) = E \left( a_i - c^i_{t+1} \right) Ed^k - \text{cov} \left( \begin{array}{c} c^i_{t+1} \\ e^i_{t+1} + \theta^f_i + \theta_i \cdot d \end{array} \right)
\]

\[
\Rightarrow \theta_i \cdot \text{cov}(d, d^k) = E \left( a_i - c^i_{t+1} \right) \left( Ed^k - \frac{q^k}{q^f} \right) - \text{cov} \left( e^i_{t+1}, d^k \right)
\]

\[
\text{where we used the pricing equation (CCAPM).}
\]

• Then, letting \( \Sigma = \text{cov}(d, d) \) we can write in matrix form:

\[
\theta_i = \Sigma^{-1} \left\{ \frac{E \left( a_i - c^i_{t+1} \right)}{E(\bar{a} - \bar{c}_{t+1})} \text{cov}(d, \bar{c}_{t+1}) - \text{cov}(d, e^i_{t+1}) \right\}
\]
An Interpretation

• Noting that:

\[ c_{t+1}^i - E c_{t+1}^i = e_{t+1}^i - E e_{t+1}^i + \theta_i \cdot (d - Ed), \]

• We have:

\[
\begin{align*}
  c_{t+1}^i - E c_{t+1}^i &= e_{t+1}^i - E e_{t+1}^i - \Sigma^{-1} \text{cov}(d, e_{t+1}^i) \\
  &\quad \left( e_{t+1}^i - E e_{t+1}^i \right) \perp \\
  &\quad + \frac{E (a_i - c_{t+1}^i)}{E (\bar{a} - \bar{c}_{t+1})} (d - Ed) \Sigma^{-1} \text{cov}(d, \bar{c}_{t+1}^i) \\
  &\quad \text{proj}(\bar{c}_{t+1}^i - E \bar{c}_{t+1}^i | d)
\end{align*}
\]

• The first term is the fraction of the agent’s risk from non financial income that is not hedgeable in the markets.

• The second term is the hedgeable fraction of her risk. It is proportional to the risk on total income and is split among agents in proportion to their risk tolerance.
Risk Free Return

- To close the model, we solve for the risk free return with the first FOC:
  \[ q^f u_t' \left( e_t^i - q^f \theta_i^f - \theta_i \cdot q \right) = \delta E \left( 1 - \frac{1}{a_i} c_{t+1}^i \right) \]

- For example, if \( u_t^i(.) = u_{t+1}^i(.) \), we can aggregate to get
  \[ q^f = \frac{\delta E (\bar{a} - \bar{c}_{t+1})}{E (\bar{a} - \bar{e}_t + \bar{\theta} \cdot q)} \]

- Note that we have implicitly assumed that all the assets were initially possessed by investors outside of the economy. If that is not the case the only thing that changes is that
  \[ q^f = \frac{\delta E (\bar{a} - \bar{c}_{t+1})}{E (\bar{a} - \bar{e}_t)} \]
CCAPM: Summary of Conclusions

• When second period non-financial incomes are certain, investors hold the same portfolio of risky securities. They vary by their total investment in the risky portfolio. (Two-Fund theorem)

• The risky portfolio is the market portfolio consisting of all the real assets.

• The risk premium of a security is proportional to its covariance with aggregate consumption.

• A portfolio of one security is most likely to be suboptimal: invest in indexes.

• CAPM is a special case in which all second period consumption comes from financial income. As a result the risk premium of a security is proportional to the covariance of its return with the market return.
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We can use the projection theorem to write for an attainable return $R^\theta$

$$R^\theta - R^f = \beta_\theta \left( R^M - R^f \right) + \varepsilon_\theta,$$

where $\varepsilon_\theta$ is orthogonal to $R^M - R^f$.

Hence

$$\text{cov}(R^M, \varepsilon_\theta) = 0$$

And by the CAPM formula $E\varepsilon_\theta = 0$.

$\varepsilon_\theta$ is the idiosyncratic part of the risk contained in $\theta$. 
Risk Decomposition

• To see that, write:

\[ \text{Var}(R^\theta) = \beta_\theta^2 \text{Var}(R^M) + \text{Var}(\varepsilon_\theta) + 2\beta_\theta \text{cov}(R^M, \varepsilon_\theta) \]

\[ = 0 \]

• The first term is the systematic risk: it cannot be eliminated through diversification, since market risk cannot be eliminated.

• The second term is security specific: this risk can be eliminated by holding many securities like \( R^\theta \) or by hedging.
Thanks!