Financial Economics
Introduction

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Ecole Polytechnique
What is Finance?

• Finance is the study of how economic agents allocate scarce resources over time and under risky conditions.

• It studies the role played by financial markets in the allocation of economic resources.

• It aims to explain what determines asset prices and interest rates.
A Simple Formal Motivation

Consider a simple model of consumption:

\( x_1 \) – Consumption of Commodity 1  
\( x_2 \) – Consumption of Commodity 2  
\( p_1 \) – Price of Commodity 1  
\( p_2 \) – Price of Commodity 2  
\( I \) – Individual Income  
\( u(x_1, x_2) \) – Utility Function of the Individual
A Simple Formal Motivation

• The consumer problem:

\[ \max_{x_1, x_2} u(x_1, x_2) \]

s.t. \[ p_1 x_1 + p_2 x_2 \leq I \]

• To solve this problem:

  • Use calculus and a simple diagram of indifference curves in order to study the optimal consumer allocation.
Optimal Consumer Allocation

• There exists a Lagrange multiplier $\lambda$ such that

$$\frac{\partial u}{\partial x_1} - \lambda p_1 = 0$$
$$\frac{\partial u}{\partial x_2} - \lambda p_2 = 0$$
$$p_1 x_1 + p_2 x_2 = I$$

• These 3 equations are solved for the three variables $(x_1, x_2, \lambda)$ as functions of $(p_1, p_2, I)$

• These are the Demand Functions.

• $\lambda$ can be interpreted as the marginal utility of income.
Finance starts from the assumption that:

- Consumption decisions are made over time: we study savings and investments.
- Savings and investments entail risks.

Hence we study the above problem over time and taking risk into account.
Some Modeling Issues

• How to define the utility of commodities over different dates?
• What is the meaning of prices \((p_1, p_2)\) of commodities over time?
• What instruments do people use to trade commodities across different dates in time?
• What is the cost of smoothing consumption over time?
• How do we quantify risk? What is the right measure?
• Can all these fit into General Equilibrium Theory?
Plan

- Lecture 1: Introduction to Finance.
- Lecture 2: Term structure of interest rates. Determinants of interest rates.
- Lecture 4: Consumption based asset pricing.
- Lecture 5: Equilibrium asset pricing. Information. A No-Trade Theorem. (HW 1)
- Lecture 6: Beta factor models, mean variance and CAPM.
- Lecture 7: Corporate Finance.
- Lecture 8: Dynamic asset pricing. Introduction to microstructure. (HW 2)
- Lecture 9: Option Pricing.
Organization of the Course
Me: Eduardo Perez-Richet

Office Hours: Tuesday 1:30-3:30pm (please e-mail first)

Phone: 01.69.33.30.43

Email: eduardo.perez@polytechnique.edu

Course Website: http://eduardo.perez.free.fr/teaching.html (syllabus, slides, problem sets, solutions, old exams and solutions)
Lectures/Sessions

• The class meets every friday from 8:30am-12:15pm.

• This time is divided between a lecture and an exercise session during which we solve some of the problem set exercises.

• The exact timing will depend on the topic.

• The slides of the lecture and corresponding problem set will be posted in advance on the website of the course.
Assignments and Exams

- **Final Exam:** Friday, November 28, 9-12am.
  - Open Book

- **Homework:** I will give two assignments that will be graded.
  - The homework can be done in groups of up to 3 students.
  - But each member of the group must write and turn in his/her own answers.
  - If you work in group, write down the names of the students you worked with.
Grading

The final grade is the best of two grades:

Grade 1 = 70% Final Exam + 25% Homework + 5% Participation

Grade 2 = 80% Final Exam + 15% Homework + 5% Participation

Your Grade = max(Grade 1, Grade 2)
Books

There is no mandatory textbook for this course. The following books are a good complementary reading, but they are by no means necessary.

First a list of graduate level textbooks.

They generally go beyond the level of this course but they are an excellent reading if you want to deepen your understanding of the theory.


Books

Second a list of MBA/undergraduate textbooks.
They are an excellent reading if you want to familiarize yourself with financial terminology, institutions and practice.

1. “Investments,” by Zvi Bodie, Alex Kane and Alan J. Marcus.


Books

Finally some popular books. Entertaining way to learn. The financial side of the story is very well described.

- Michael Lewis, “Liar’s Poker”
- Michael Lewis, “The Big Short”
The Financial System
The Financial System

Financial Intermediaries:
- Banks
- Credit Unions
- Brokers
- Insurance
- Mutual Funds
-...

Government

Business

Non-Residents

Households
Financial Intermediaries

• The financial intermediaries channel funds from households (savings, pensions) to businesses and governments.

• For example, banks take deposits, pool them and use them to issue loans and mortgages to businesses and households.

• Insurers sell policies and collect premiums, they invest the premiums so that the accumulated value grows and can meet the anticipated claims of the policyholders. Allows risk sharing.

• Mutual funds give small investors access to diversified, professionally managed portfolios of securities.

• Pension funds collect pension payments and invest these funds.

• More generally, they issue securities and invest in securities. Also repackaging, financial innovation etc.
Financial Assets
Real vs. Financial Assets

An asset is an economic resource. Anything tangible or intangible that can be owned or controlled to produce value and that is held to have positive economic value is considered an asset. (Wikipedia)

- **Real Assets**
  - Assets used to produce goods and services.
  - Examples: factories, land, human capital etc...

- **Financial Assets**
  - Contracts for delivery of some income streams (e.g. bonds)
  - Claims on the benefits from ownership of real assets (e.g. stocks)
  - Derivatives

- Property rights and the force of law protect these contracts or claims.
Use of Financial Assets

• Financial assets contribute to economic efficiency.

• They allow people to time their consumption separately from their labor or other income (savings instruments).

• They allow economic agents to allocate risk (diversification, hedging, insurance).

• With the help of competition, financial markets channel capital from savers to desirable investment projects.
Use of Financial Assets

• They allow the separation of ownership and management. This permits the existence of large companies yet allows for distribution of ownership.

Example: Ownership of Google in September 2012 (from Yahoo! finance)

• Top Institutional Holder: FMR LLC (Fidelity Investment). 16,540,629 shares. 6.51%
• Major Direct Holder: Kavitark Ram Shriram. 148,602 shares. <1%
Important Financial Assets

• Debt or Fixed Income Securities: promises to pay future money.

• Equity: ownership of a firm, stock – residual claims on assets of Limited Liability Corporations.

• Derivatives: securities whose payoffs depend on the value of other assets. Options, futures, swaps.

• Investment Companies: Mutual funds.
Classification by Maturity

- **Money Market Instruments**: short-term, liquid, free of default risk. e.g. Treasury Bills (US), BTF (France), commercial papers, short term time deposits, money funds, short lives mortgage and asset-backed securities etc..

- **Capital Market**: long term, includes the bond market and the stock market. e.g. Equity (various categories of stocks), US treasury Notes, US Treasury bonds, OAT, BAN (France), Corporate Debt, Derivatives etc...
Financial Markets
Financial Markets

• **Primary market:**
  - Issuance of new securities by companies, government or public sector institutions.

• **Secondary Market:**
  - Where previously issued securities are traded.
  - **Exchanges or auction markets:** organized market in a specific place and a specific bidding process.
  - **Over The Counter markets:** network of dealers who trade directly with one another. Derivatives are often traded this way.
Financial Markets

• Third market:
  • Trading of exchange-listed securities in the OTC market.

• Fourth Market:
  • Direct trading between investors (usually big institutions) without the involvement of brokers or dealers.
Fixed Income Securities
Interest Rates and Rates of Return

Example: US Treasury Bills

- Traded in denominations of $10,000; maturity of 13, 26 or 52 weeks.
- Sold at discount: you pay, say, $9,920 for a 26 weeks T-bill promising to pay you at maturity the face value of $10,000.
- What is the effective annual yield?
  - 6-month return: \( \frac{80}{9,920} = 0.0080645 \) or 0.80645%.
  - Effective Annual Yield: \((1 + 0.0080645)^2 - 1 = 0.01619 = 1.619\% \)
  - Note that the published yield is often wrong: \( \frac{80}{9,920} \times \frac{365}{182} = 1.617\% \)
Coupon and Zeros

- Most bonds carry a **coupon rate** which is a fixed nominal amount paid to the holder every 6 month until maturity.

- A **zero coupon bond** is an obligation to pay a fixed nominal amount at maturity.

- Example: a 20 year bond with face value $1,000 and a coupon rate of 4% pays the following cash flow:

  \[
  \begin{array}{cccccc}
  \$20 & \$20 & \cdots & \$20 & \$1,020 \\
  \end{array}
  \]

  with a payment every 6 months for 20 years, hence 40 payments.
Market Created Zero Coupon Bonds

- US Treasury STRIPS (Separate Trading of Registered Interest and Principal of Securities).

- With permission of the US treasury, traders strip bonds: they reissue each of the individual coupon and principal payments as a zero coupon bond with diverse maturities.

- For example, the 20 year bond we looked at is stripped into 41 zero coupon bonds.
Default Risk

- Default is the event in which some portion of the interest or the principal of a fixed income security cannot be paid.

- The risk of default is an important component of private debt.

- The rating of corporate bonds reflects their default risk and will be related to their return.

- For example, the bonds rated lower than BB by S&P are considered junk bonds.
The Yield Curve

• Usually, national debt securities have no default risk.

• But their market interest rates vary by maturity.

• Typically long term rates are higher than short term rates.

• Bond prices are quoted as a percentage of par value.
  • A 5% coupon, 20 year bond with face value $1,000 priced at 106.75 means that it costs $1,067.5.
The Yield Curve


<table>
<thead>
<tr>
<th></th>
<th>Coupon</th>
<th>Maturity</th>
<th>Price</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-month</td>
<td>0.00</td>
<td>12/12/2011</td>
<td>99.89</td>
<td>0.52</td>
</tr>
<tr>
<td>6-month</td>
<td>0.00</td>
<td>12/3/2012</td>
<td>99.76</td>
<td>0.53</td>
</tr>
<tr>
<td>2-year</td>
<td>4.50</td>
<td>7/3/2013</td>
<td>105.83</td>
<td>0.55</td>
</tr>
<tr>
<td>5-year</td>
<td>4.00</td>
<td>7/9/2016</td>
<td>113.41</td>
<td>1.22</td>
</tr>
<tr>
<td>10-year</td>
<td>3.75</td>
<td>7/9/2021</td>
<td>112.02</td>
<td>2.39</td>
</tr>
<tr>
<td>30-year</td>
<td>4.25</td>
<td>7/12/2040</td>
<td>112.13</td>
<td>3.58</td>
</tr>
</tbody>
</table>

(The yield is a measure of returns.)
The Yield Curve
Stocks
Stocks

- Stocks are securities with limited liability.
  - In case of default, no one can go after the stockholders.

- A stock is a residual claim on the assets of a corporation: it is a claim on whatever is left after employees, contractors and debt holders have been paid.

- The aggregate value of stocks is very large. For example at the end of 1998 the aggregate value of different assets in the US:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>$15.4 Trillion</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>$3.9 Trillion</td>
</tr>
<tr>
<td>Treasury Securities</td>
<td>$3.7 Trillion</td>
</tr>
</tbody>
</table>
Stock Returns

• There are two types of returns on stocks:
  • Dividends – cash payments from the profits of corporations.
  • Capital Gains – change in market value.

• Example: you pay $75 for a stock that you hold for a year.
  • It pays a dividend of $4 during the year
  • You sell it for $80.

• Your return is: \( \frac{80 - 75 + 4}{75} = 12\% \)

• If instead the stock price had declined to $65, your return would be: \( \frac{65 - 75 + 4}{75} = -8\% \)
Market Indexes

• A market index measures the performance of a section of the stock market.

• Examples:
  • US: DJIA, SP500, NASDAQ.
  • Japan: Nikkei
  • Europe: CAC 40, FT-30, FT-100.

• These indexes measure whether “the market” is up or down.

• They can track the long term performance of stocks vs. other financial assets.
Dow Jones Industrial Average

• Index of 30 large companies.

• Composition changes over time.

• The index is not weighted: it simply sums the prices of the stocks adjusting the denominator (DIVISOR) for stock splits and changes in the composition.
  
  • Example: initially 2 stocks: \( S_1 = 25, S_2 = 100 \).

\[
DJ = \frac{S_1 + S_2}{2} = 62.5
\]

• Stock 2 splits, no change in total market value but there are now two shares for company 2. Now \( S_1 = 25, S_2 = 50 \)

\[
DJ = \frac{S_1 + S_2}{d} = 62.5
\]

• So \( d = 1.2 \)
Others

• **SP500**: index of 500 large companies, it is weighted by market-value and is a better representation of the market.

• **NASDAQ**: index of 4,000 OTC traded companies. Market-value-weighted. It better represents new and younger companies.

• **CAC 40**: Index of 40 large French companies. Market-value weighted.
Stock Split Example

• Example: initially 2 stocks, one for each company: 
  \[ S_1 = 25, S_2 = 100. \]

  \[ \text{CAC 40} = S_1 + S_2 = 125 \]

• Stock 2 splits, no change in total market value but there are now the price of a stock in company 2 is divided by 2. Now 
  \[ S_1 = 25, S_2 = 50 \]

  \[ \text{CAC 40} = S_1 + 2S_2 = 125 \]
Composition of the CAC 40


ACCOR, AIR LIQUIDE, ALCATEL-LUCENT, ALSTOM, ARCELORMITTAL, AXA, BNP PARISBAS ACT A, BOUYGUES, CAP GEMINI, CARREFOUR, CREDIT AGRICOLE, DANONE, EADS, EDF, ESSILOIR INTL, FRANCE TELECOM, GDF SUEZ, LAFARGE, L’OREAL, LVMH, MICHELIN, NATIXIS, PERNOD RICARD, PEUGEOT, PPR, PUBLICIS GROUPE SA, RENAULT, SAINT GOBAIN, SANOFI-AVENTIS, SCHNEIDER ELECTRIC, SOCIETE GENERALE, STMICROELECTRONICS, SUEZ ENVIROMNEMENT, TECHNIP, TOTAL, UNIBAIL-RODAMCO, VALLOUREC, VEOLIA ENVIRONNEMENT, VINCI, VIVENDI
Evolution of Stock Value 1950-2010

Dow Jones Industrial Average

Source: Yahoo! Finance
Evolution of Stock Value 1979-2011

## Returns of Stocks


<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Stocks</td>
<td>13.2%</td>
<td>20.3%</td>
</tr>
<tr>
<td>Small Stocks</td>
<td>17.4%</td>
<td>33.8%</td>
</tr>
<tr>
<td>LT corp bonds</td>
<td>6.1%</td>
<td>8.6%</td>
</tr>
<tr>
<td>LT Gov Bonds</td>
<td>5.7%</td>
<td>9.2%</td>
</tr>
<tr>
<td>IT Gov Bonds</td>
<td>5.5%</td>
<td>5.7%</td>
</tr>
<tr>
<td>T Bills</td>
<td>3.8%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Inflation</td>
<td>3.2%</td>
<td>4.5%</td>
</tr>
</tbody>
</table>

Why are the returns so different?
## Returns of Stocks

**Summary Statistics of Inflation Adjusted Annual Total Returns, US 1926-1998**

<table>
<thead>
<tr>
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<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Stocks</td>
<td>9.9%</td>
<td>20.4%</td>
</tr>
<tr>
<td>Small Stocks</td>
<td>13.9%</td>
<td>33.1%</td>
</tr>
<tr>
<td>LT corp bonds</td>
<td>3.1%</td>
<td>9.9%</td>
</tr>
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</tr>
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<td>4.1%</td>
</tr>
</tbody>
</table>

Because riskier assets demand higher returns. But when looking at portfolio holdings, the only explanation for people holding so little stock over this period is an improbable level of risk aversion. This discrepancy between the data and the theory is called the equity premium puzzle.
Returns of Stocks

• If you invested $1 in the SP500 index on January 1, 1926, it was worth $2,642 on July 31, 1999.

• Adjusting for inflation you would have $283.93.

• If instead you had invested in US government LT bonds, you would have $4.44.

• This is an underestimate since it does not take dividends into account.
Trading Stocks

• Cost of trading:
  • Commissions.
  • Bid-Ask spread: the Bid is the price at which the dealer will buy from you, the Ask is the price at which he will sell to you.

• Different types of order:
  • Market orders: buy or sell at the current market price.
  • Limit orders: buy at no more or sell at no less than a specified price.
  • Stop Loss orders: sell if the price goes below, or buy if the price goes above a specified price.
Trading Stocks

• Different trading procedures:
  • NYSE - specialist.
  • NASDAQ - computer bidding.

• **Buying long**: owning stock, hoping that its price goes up.

• **Buying on Margin**: borrowing money from a broker to buy stock.
  • The margin requirement is set by the Fed. Currently: 50%

• **Selling short**: The seller borrows the stock from a lender and sells it on the market only to buy it back later and return it to its owner (hoping that the stock went down).
Short Selling Procedure

- You need a margin account to short sell stocks.
  - You borrow the stock from the broker.
  - You deposit a margin requirement.
  - You sell the stock on the open market.
  - When you close your position, you use the funds at the broker to repurchase and return the borrowed stock. You keep the balance.
Example

• You borrow 1,000 shares of Yahoo! and sell them at the market price of $24.

• $24,000-$250 (commission)=$23,750.

• This amount is kept by the broker, as well as a 50% margin of $12,000 worth of assets.

• Suppose Yahoo! falls to $20. You repurchase all the shares for $20,000+$225 (commission). Your net profit is $3,525.

• Yahoo! could have gone up to $100 and you would have lost more than $76,000.

• Different than buying a stock. Your losses are unbounded.

• In practice the margin account allows the broker to close your position when it becomes too dangerous.

• Dividends: you must pay the dividends of the stock to the lender.
Derivatives
Derivatives

- What is the use of derivatives?
- **Hedging**: a position intended to offset potential losses.
  - A farmer can hedge the risk associated with changes in the price of cereals.
  - A gas company (change in the buying price of gas).
  - A multinational firm hedges the exchange rate risk.
- **Speculation**: no clear definition, basically involves a more risky bet than an investment.
  - Derivatives are liquid and often less regulated.
  - They can be used to create leverage.
Futures and Options

- **Futures**: a contract to exchange of a specified quantity of a good at a specified date in the future for a price agreed today.

- **Call Option**: gives you the right to buy an asset at specified price over a specified period of time.

- **Put Option**: gives you the right to sell an asset at a specified price over a specified period of time.

- **European options**: can only be exercised at the maturity date.

- **American options**: can be exercised at any time before the maturity date.
Swaps

- **Swap**: an agreement between two parties to exchange the benefits of their respective financial instruments.

- The two instruments are called the legs of the swap.

- For example in a Credit Default Swap, the buyer makes a series of payments to the seller, while the seller makes a payment to the buyer if a certain fixed income security goes into default.
Example: Credit Default Swap

- $B$ holds some long maturity French treasury bonds.

- To protect herself against default, $B$ purchases a CDS on French bonds.

- She must make a payment to the seller of the CDS every six months.

- If there is a default, the seller of the CDS makes a payment to $B$.

- So a CDS is an insurance against default (useful).

- But it can also be used to bet on a default by an investor who does not hold French bonds.

- This bet is also achievable by shorting French bonds, but for this you need to invest more funds. Derivatives allow more leverage.
Asset-Backed Securities

- An asset-backed security is the bundling of existing securities such as:
  - Mortgages
  - Auto loans
  - Student loans
  - credit card receivables etc.

- An example of financial engineering.
The Theory of How the Financial System Created AAA-rated Assets out of Subprime Mortgages

In the financial system, AAA-rated assets are the most valuable because they are the safest for investors and the easiest to sell. Financial institutions packaged and re-packaged securities built on high-risk subprime mortgages to create AAA-rated assets. The system worked as long as mortgages all over the country and of all different characteristics didn't default all at once. When homeowners all over the country defaulted, there was not enough money to pay off all the mortgage-related securities.

1. People all over the country take out mortgages. Financial institutions group hundreds of subprime mortgages into Mortgage Backed Securities (MBSs).

2. The securities are grouped into tranches by levels of risk and earnings potential for bond holders. When everybody can pay their mortgage in full each month, each group of bond holders gets paid.

3. The mortgage payments are collected by a financial institution and payments distributed to bond holders. Higher-rated tranches are paid first. When monthly mortgage payments are not made, payments may not reach holders of lower-rated tranches.

4. Collateralized Debt Obligations (CDOs) were created by taking the lower-rated tranches out of the MBSs and repackaging them. Most of this CDO is highly rated, even though it is built out of high-risk assets.

5. Another financial institution does the same thing with high-risk tranches of CDOs, creating a CDO-squared.

Asset Pricing
Asset Prices

• What determines interest rates and asset prices?
  • Current dividends?
  • Not the dividends: if profits are reinvested, current dividends should not matter.
  • High present dividends do not imply high stock prices.
  • Book value: often misleading, does not reflect the ability of the firm to earn future profits.

• What really matters is the productivity of the firm and the implied flow of future dividend payments.
Questions

• How to evaluate future payments?
• How does the market evaluate these future payments?
• How is this reflected into prices?
• How is risk incorporated into prices?
Deterministic Cash Flows
The Ideal Bank

- Imagine a bank that charges no fees.

- Constant interest rate over time \( r \):
  - Deposit $1 today and you have \((1 + r)\) after one year.
  - Borrow $1 today and you have to pay \((1 + r)\) in a year.

- For example, if \( r = 10\% \), $150 today are worth $165 in a year.

- If you leave the money one additional year you have

\[
165 + 16.5 = $181.5
\]
Compounding

- The interest can be reinvested and then grows at the same rate as the principal.

- Hence after $T$ years, your principal is multiplied by $(1 + r)^T$:

\[
FV = PV \times (1 + r)^T
\]

- This is the “fundamental law of finance”
How fast does it grow?

- Doubling time

\[(1 + r)^T = 2 \iff T = \frac{\log 2}{\log(1 + r)} \sim \frac{0.7}{r}\]

- At \( r = 7\% \) your deposit doubles in approximately 10 years.

- $1 invested at 7\% for 100 years gives $868

- $1 invested at 10\% for 100 years gives $13,781
Compounding at shorter intervals

- Suppose the ideal bank pays interests quarterly at rate $r/4$.
- These interests can reinvested so that the effective yearly return is
  \[(1 + r/4)^4 > (1 + r)\]

- For example, with $r = 12\%$,
  \[(1 + .03)^4 = 1.1255 > 1.12\]

- Sometimes, deceptive advertising. If a loan has a 6% APR rate and is in fact payable monthly, the effective interest rate is 6.1678\%.
Continuous Compounding

- If compounding $n$ times per year,

$$1 + r_e = (1 + r/n)^n$$

- At the limit, with continuous compounding

$$1 + r_e = \lim_{n \to \infty} (1 + r/n)^n = e^r$$

- Return after $t$ years: $e^{rt}$

- The deposit (or debt) grows exponentially.

- Then the doubling time is exactly equal to $\frac{\log 2}{r}$. 
Suppose you consider buying an apartment as an investment, so you want to know how much it is worth.

Future cash flows are rents you receive (if you live in the apartment, they are the rents you do not have to pay).

As an approximation, assume current rent payment will continue for ever.

How to evaluate your investment? What is the present value of this stream of future cash flows?
Present Value

• If you know for sure that you will get $100 in a year, but you need money today.

• You can borrow money today that you will repay with your $100 cash flow in a year.

• The amount you can borrow today is

\[ PV = \frac{100}{1 + r} \]

• If instead you know that you will receive $100 in two years, today it is worth

\[ PV = \frac{100}{(1 + r)^2} \]
Present Value

• The present value of a certain stream of cash flows \((x_1, x_2, \cdots, x_T)\) today is:

\[
PV = \sum_{t=1}^{T} \frac{x_t}{(1 + r)^t}
\]

• For a constant cash flow \(x\) over \(T\) years

\[
PV = x \left( \frac{1}{r} - \frac{1}{r(1 + r)^T} \right)
\]

• For a constant cash flow \(x\) that lasts for ever

\[
PV = x \sum_{t=1}^{\infty} \frac{1}{(1 + r)^t} = \frac{x}{r}
\]
Examples

• Suppose you win $20 millions at the Euromillion, but it is paid over 20 years and $r = 10$

\[ PV = $8.514 \text{ millions} \]

• Suppose someone offers to pay you $1 every year and for ever, with $r = 5$

\[ PV = $20 \]
Evaluating Projects

Fig. 2. Survey evidence on the popularity of different capital budgeting methods. We report the percentage of CFOs who always or almost always use a particular technique. IRR represents internal rate of return, NPV is net present value, P/E is the price-to-earnings ratio, and APV is adjusted present value. The survey is based on the responses of 392 CFOs.

Net Present Value

- **NPV** = the difference between an investment’s market value (the present value of the stream of cash flows it generates), and its cost.

- **Rule:** Accept if \( NPV > 0 \)

- **Example:**
  - Your (riskless) firm is worth 3M: 1M in cash and 2M in stocks.
  - There are 0.5M shares outstanding. The return on the stock is 12%.
  - You have a (riskless) investment opportunity costing 1M and offering cash flows of 0.3M for five years.
  - Should you undertake this investment?
The NPV of the investment is:

\[
NPV = -1M + \left( \frac{1}{0.12} - \frac{1}{0.12 \times 1.12^5} \right) \times 0.3M = 81,432 > 0
\]

You should undertake the investment.

What will happen to the stock price?

Before: \( p = \frac{3M}{0.5M} = 6 \)

After: \( p' = \frac{3M + 0.081M}{0.5M} = 6.16 \)

What if the return on the stock was 17%?

\( NPV = -40,196 \) and \( p' = 5.92 \)
NPV: Conclusion

• The main assumption here is that there exists a market interest rate at which the firm can borrow as much as it wants.

• This criterion suggests a firm borrows as much as needed for all projects with positive NPV.

• This ignores uncertainty. The method is also used with uncertainty but then the question is what interest rate should be used to calculate the present value.
Internal Rate of Return

- **IRR**: the interest rate that makes the NPV equal to 0.

- **Rule**: Invest if IRR is greater than the cost of borrowing $r$.

- **Interpretation**: IRR measures the internal growth rate of the project.
IRR: Example

- Consider the same investment project as before. The IRR solves

\[-1 + 0.3 \left( \frac{1}{x} - \frac{1}{x(1 + x)^5} \right) = 0\]

- Solve \(\Rightarrow IRR = 15.4\%\).

- Hence if the cost of borrowing is 12% you should invest whereas if it is 17% you should not.

- Advantage: the IRR allows you to compare projects without knowing the cost of borrowing.
IRR vs NPV

• Suppose you plant a tree that costs $10 and you can have the choice to cut it after one or two years.

• After one year you get $20 worth of wood, while after two years you get $30.

• Suppose the cost of borrowing is 10%. What should you do?

\[ \text{NPV}_1 = -10 + \frac{20}{1.1} = 8.2 \quad \text{IRR}_1 = 100\% \]

\[ \text{NPV}_2 = -10 + \frac{30}{1.1^2} = 14.3 \quad \text{IRR}_2 = 73.2\% \]
Time Consistency

• Do you make the same decision after one year?

• Consider the decision of letting the tree grow another year when you reach the end of the first year. This investment has an opportunity cost of $20 and generates a cash flow of $30 in one year.

\[ NPV_1 = \frac{30}{1.1} = 27.28 \]

\[ IRR_1 \text{ solves } -20 + \frac{30}{1 + x} = 0 \]

\[ \Rightarrow IRR_1 = 50\% > 10\% \]

• NPV is a time consistent criterion while IRR is not!
IRR vs NPV 2

- There are some other pitfalls with IRR:
  - It ignores the magnitude of the project (IRR is invariant to scale).
  - The equation of IRR may have multiple solutions.
  - Projects with opposite cash flows have the same IRR!

- It is still a good measure of the growth potential of a project, but it should be used carefully as a decision rule.
Thanks!